

1. Calculate the density of states in 2D and 1D for parabolic bands. Explain why it is alright to take one of the limits to be infinity in equations that give the total number of electrons (holes) even though all bands have finite extents. Can you estimate the typical error due to this approximation at $T=300\text{K}$?
2. Show that the deviation of the electron density (n) from intrinsic density (n_i) and the deviation of the Fermi level (E_f) from the intrinsic Fermi level (E_{fi}) are related as :

$$n = n_i e^{\beta(E_f - E_{fi})}$$

3. Show that in a 2-dimensional semiconductor, the "intrinsic" level will be located at:

$$E_{fi} = \frac{E_C + E_V}{2} + \frac{1}{2} k_B T \ln \frac{m_h}{m_e}$$

4. Show that the carrier density in a semiconductor with N_D donors, can be obtained by solving the following equation:

$$n^2 + n N_C \frac{e^{-\beta\Delta}}{2} - N_D N_C \frac{e^{-\beta\Delta}}{2} = 0$$

where $\Delta = E_C - E_D$.

The fermi level can be obtained by solving

$$x^2 + x \frac{e^{-\beta\Delta}}{2} - \frac{N_D}{N_C} \frac{e^{-\beta\Delta}}{2} = 0$$

where $x = e^{\beta(E_f - E_C)}$ and N_C is the effective conduction band density of states.

- (a) If you put $N_D = 0$ in either of the two equations you would get an unphysical answer. Why is this so?
 - (b) For a practical doping density like $N_D \sim 10^{17} \text{cm}^{-3}$, estimate the error are you introducing by ignoring the (intrinsic) carriers excited from the valence band. Take the case of Silicon at $T=300\text{K}$ to be specific. Would this error be larger for Germanium, Diamond?
5. In a system with N_D donors, N_A acceptors, N_D^+ donors and N_A^- acceptors are ionised. Each donor (acceptor) level has a degeneracy of g_D (g_A). There are n electrons in the conduction band and p holes in the valence band. (In general $g_D = 2$, but g_A may be different from 2.). Then

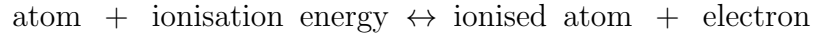
$$N_D^+ = \frac{N_D}{(g_D n / N_C) \exp \beta(E_C - E_D) + 1}$$

And the corresponding result for the acceptors:

$$N_A^- = \frac{N_A}{(g_A p / N_V) \exp \beta(E_A - E_V) + 1}$$

Here N_C and N_V are the conduction and valence band effective density of states which have been defined earlier. Notice that the fermi energy does not appear in these relations.

6. Ionisation of a donor can be thought as a "chemical" reaction:



A certain fraction of atoms will exist in the dissociated state and a certain fraction will remain in the undissociated state. The fraction which minimises the free energy of the entire system (at a certain temperature) will fix the equilibrium point.

Taking this approach we can calculate the ratio N_D^+/N_D by minimising the free energy of the entire system of free electrons and the dopants. First write the free energy so that the free electron concentration n is the only variable.

$$\begin{aligned} F_{system} &= F_{electrons} + F_{dopants} \\ F_{electrons} &= -kT \ln \frac{z^n}{n!} \text{ where for a single electron} \\ z &= \sum_{\text{all states}} e^{-\beta E} \\ &= V \frac{2}{h^3} \int d^3 \mathbf{p} e^{-\frac{\beta p^2}{2m}} \\ &= 2V \left(\frac{2\pi m k T}{h^2} \right)^{3/2} \end{aligned}$$

Now since $N_D - n$ dopant sites are occupied we have for the internal energy (U) and entropy (S)

$$\begin{aligned} U &= -\Delta(N_D - n) \\ S &= k \ln \left(2^{N_D - n} \frac{N_D!}{n!(N_D - n)!} \right) \\ F_{dopants} &= U - TS \end{aligned}$$

Minimise $F_{system} = F_{electrons} + F_{dopants}$ w.r.t n , using Stirling's approximation for factorials as needed and show that you get exactly the same result as eqn as the last problem. This is essentially a variant of the "Saha ionisation" equation, applied to a situation where the atoms and ions are not mobile, but only the electrons are.