

**EP431: Semiconductor Physics: More practice problems.....**

1. A 2 dimensional lattice is defined by the two vectors, where  $a$  is some length

$$\mathbf{a}_1 = a \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right)$$

$$\mathbf{a}_2 = a \left( -\frac{1}{2}, \frac{\sqrt{3}}{2} \right)$$

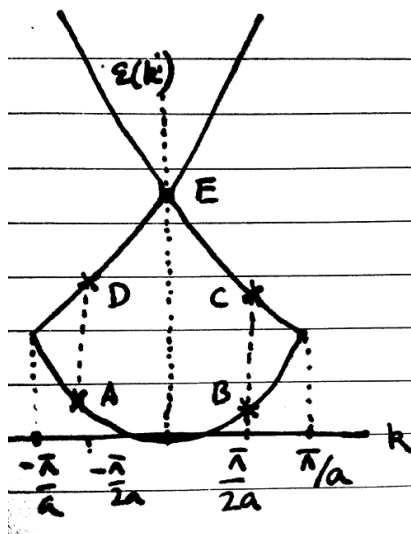
Calculate the two reciprocal lattice vectors.

2. An electron, of charge  $-|e|$  moves in a (2-dimensional) band whose dispersion relation is given by

$$E(k_x, k_y) = \frac{\hbar^2}{2m_0} (3k_x^2 + 5k_y^2)$$

An electric field  $\mathbf{F}$  is applied in a direction  $45^\circ$  to the  $k_x$  axis. What will be the angle between the direction of acceleration of the electron and the electric field? *Your answer must be a single number, e.g.  $8^\circ$*

3. Consider a 1-d chain of atoms of lattice constant  $a = 1\text{nm}$ , in which an electron moves under the influence of an electric field  $F = 10\text{V.m}^{-1}$ . The relaxation time in the system due to all sources of scattering is  $\tau = 10^{-14}\text{sec}$  which is typical of many substances. If the electron was originally at  $k = 0$ , what fraction of the Brillouin zone will it be able to explore in its course of motion?
4. Consider an 1-dimensional band of nearly free electrons, plotted in the commonly used reduced zone scheme, as shown in the figure. Write down the energy and unnormalised wavefunctions corresponding to the points marked A,B,C,D,E.



5. The unit vectors of a 2D lattice are  $\mathbf{a}_1$  and  $\mathbf{a}_2$ . The included angle between them is  $\theta$ . For convenience take  $\mathbf{a}_1$  to be along  $x$  axis. Calculate the reciprocal vectors  $\mathbf{b}_1$  and  $\mathbf{b}_2$  explicitly. What are the possible angles between the reciprocal vectors?
6. Consider a finite chain of 100 identical atoms spaced by  $5\text{\AA}$ .
- What is the linear span of the first Brillouin zone? Give your answer in units of  $\text{cm}^{-1}$ ?
  - The modulus of the quantity  $S(k) = \sum_n \exp(ikx_n)$  is defined as the Structure factor, where  $x_n$  is the position of the  $n^{\text{th}}$  atom. What are the location and width of the first peak of  $|S(k)|^2$   $k \neq 0$ , in  $\text{cm}^{-1}$ ?

Your answers must be explicit numbers.

7. Consider an one dimensional band whose dispersion relation is  $E(k) = E_0 - 2\gamma \cos ka$ , where  $\gamma$  is a measure of the nearest neighbour interaction and  $a$  is the lattice constant. An electron driven by an electric field  $F$  moves in this band without getting scattered.

- (a) Find an expression for the amplitude of Bloch oscillations in this band.  
 (b) For a bandwidth of 1 eV, and a realistic electric field 10V/m, calculate the amplitude. How does this length compare to typical mean free paths in a very clean metal at low temperature - say 500 microns?

8. A 2 dimensional system has a dispersion relation

$$E(k_x, k_y) = \frac{\hbar^2}{2m_x} k_x^2 + \frac{\hbar^2}{2m_y} k_y^2$$

Find an *explicit* algebraic relation for the density of states function  $D(E)$ . Your answer should be free of any integrals etc. You may ignore the spin degeneracy.

9. A hypothetical 2-dimensional substance  $X$  can freeze in two forms - a triangular lattice with lattice constant  $a$  and a square lattice with the same lattice constant. Consider the band arising out of the outermost spherically symmetric  $s$  state. In which case would you expect the bandwidth to be larger? The credit is entirely for giving the correct reasoning.
10. Electrons in an one-dimensional band have an energy dispersion given by:

$$E(k) = E_0 - 2|t| \cos ka$$

Where  $E_0$ ,  $t$  and  $a$  are constants,  $k$  is 1-dimensional and restricted to  $-\pi/a < k < \pi/a$ . You can ignore spin degeneracy.

- (a) What is the sign of the effective mass near  $k = 0$  and  $k = \pm\pi/a$ ?  
 (b) Calculate the co-ordinate of the point where the effective mass changes sign.  
 (c) Suppose the band is completely full. Calculate the explicit value of the sum total of all group velocities, *i.e.* evaluate

$$\frac{L}{2\pi} \int_{\text{all states}} v_g dk$$

- (d) Derive an explicit expression for the density of states  $D(E)$  and make a rough sketch with the axes clearly labelled.

11. The two topmost bands of Graphene can be described by the dispersion relations:

$$E(k_x, k_y) = \tilde{E}_0 \pm t|F(k_x, k_y)|$$

where  $F(k_x, k_y) = \left(1 + 2 \cos\left(\frac{k_x a}{2}\right) e^{-i\frac{\sqrt{3}}{2} k_y a}\right)$

Here  $t$  is the nearest neighbour hopping matrix element and  $a = 2.46\text{\AA}$  is the lattice constant.

- (a) Find the co-ordinates of the set of *all* points  $(k_x, k_y)$  at the boundary of the first Brillouin zone at which the two bands touch each other.  
 (b) Near one of these points the dispersion relation gives  $E(\mathbf{k}) = \hbar v|\mathbf{k} - \mathbf{k}_0|$  where  $\mathbf{k}_0$  is one such point. If  $v \approx 10^6 \text{ m.s}^{-1}$  estimate the value of  $t$  in electron volts.

(12 + 8 marks)

12. For this problem take  $\frac{\hbar^2}{2m} = 1$  to simplify the algebra, such that for free electrons  $E(k) = k^2$ . Consider a 1-d lattice with a lattice constant  $a$  and a potential

$$V(x) = -2V_0 \cos \frac{2\pi}{a} x$$

- (a) Consider the region close to the zone boundary at  $\frac{\pi}{a}$ . The electron states around this point may be written as a superposition of two plane wave states

$$\psi(x) = Ae^{ikx} + Be^{i(k-G)x}$$

where  $G = \frac{2\pi}{a}$ . Formulate the set of equations necessary to solve for the unknown coefficients  $A$  and  $B$ . Solve for the eigenfunctions and show that

$$E(k) = \left(k - \frac{G}{2}\right)^2 + \frac{G^2}{4} \pm \sqrt{G^2 \left(k - \frac{G}{2}\right)^2 + V_0^2}$$

- (b) Now suppose we want to solve an inverse problem where the energy  $E$  is given and you have to find  $k$ . If  $E = \frac{G^2}{4}$  and the lattice potential is non-zero, what can you say about the possible values (solutions) of  $k$ ? Your answer should be a single clear mathematical statement - *not a long-winded explanation*

13. The Fermi integrals often appear in the calculation of quantities that require an integration over the full set of particles with the (thermodynamic) Fermi distribution. It is defined as

$$F_{\frac{1}{2}}(z) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} dx \frac{x^{1/2}}{1 + e^x e^z}$$

- (a) Approximate  $F$  when  $z$  is large and negative  
 (b) Approximate  $F$  when  $z$  is large and positive  
 (c) Answer with reason whether  $F(z)$  has a maxima or minima. Using these three information make a qualitative sketch of  $\ln F(z)$  vs  $z$
14. Certain combinations of the Fermi function, occur very frequently in expressions that involve scattering or transitions. Consider the equilibrium Fermi distribution

$$f = \frac{1}{e^{(E-E_F)/kT} + 1}$$

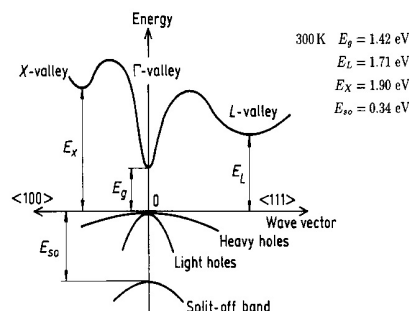
It is useful to be familiar with the combination  $f(1-f)$

Make a rough sketch of how  $f(1-f)$  would look as a function of energy. How does the area under the curve of  $f(1-f)$  vary with temperature?

15. A sample of Silicon is doped with  $N_D = 10^{17} \text{cm}^{-3}$  of Arsenic. The donor level is 54meV below the conduction band bottom. Ignore the presence of the valence band and assume that all electrons come from the donors only. Use the non-degenerate approximation for carrier densities. The effective conduction band density of states for Si at 300K is  $N_C = 2.8 \times 10^{19} \text{cm}^{-3}$ . At  $T = 300\text{K}$  calculate:
- (a) What fraction of the dopants are ionised? *Do not assume all donors will ionise! Calculate the fraction.*  
 (b) the location of the Fermi level ( $E_F$ ) w.r.t the bottom of the conduction band. How far is it from the donor level? Was the non-degenerate approximation justified?

Your answers must be in numbers - not algebraic expressions.

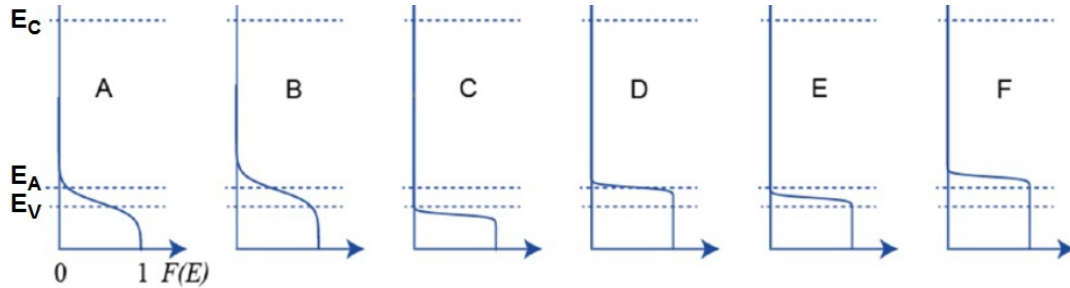
16. The highest mobility achieved so far in a semiconductor is  $\mu = 3 \times 10^3 \text{m}^2 \text{V}^{-1} \text{s}^{-1}$  at a carrier concentration  $n = 2 \times 10^{15} \text{m}^{-2}$ . This was achieved in a TWO dimensional electron gas in Gallium Arsenide, at very low temperatures. (The temperature/material are not important in this problem, these are just for information.) Calculate the mean free path ( $l$ ) under these circumstances and compare it with the lattice constant ( $a \sim 5\text{\AA}$ ) of Gallium Arsenide? What is the approximate value of  $l/a$ ?
17. The figure below shows the band structure of a semiconductor. Notice the two conduction band minima at the  $\Gamma$  point and the  $L$  point. The effective masses at these two points are  $0.066m_0$  and  $0.55m_0$  respectively where  $m_0$  is the free electron mass.



- (a) Is this a direct or indirect gap material?

- (b) At high electric fields electrons from  $\Gamma$  valley may be excited to the  $L$  valley. How do you expect the conductivity to be affected? Answer with reason.
- (c) Is the density of states higher near  $\Gamma$  valley bottom or the  $L$  valley bottom? Answer with reason.
- (d) Make a qualitative plot of the velocity of the electrons  $v(k)$  with the  $\Gamma$ - $L$  direction (in  $k$  space) on the  $x$ -axis.

18. The figure below shows relative locations of the conduction band bottom ( $E_C$ ), acceptor level ( $E_A$ ) and the valence band edge ( $E_V$ ). Answer with very brief (one sentence) reasoning.  $F(E)$  denotes the probability of occupation of the level.



- (a) Which one(s) of the six figures may be realised in an extrinsic p-type semiconductor in practice? Answer with *very* brief reason.
- (b) In which case would one have the maximum *electron* density?
19. The bandgap of Si (in eV) varies with absolute temperature according to the following relation

$$E_g(T) = 1.17 - 4.73 \times 10^{-4} \frac{T^2}{T + 636}$$

Calculate the concentration of electrons ( $n$ ) in the conduction band of intrinsic Si at 77 K, if it is known that at 300K  $n = n_i = 1 \times 10^{10} \text{cm}^{-3}$

20. Consider particle of mass  $m$  in a "toy - molecule" potential given by

$$V(x) = -|V_0|L [\delta(x) + \delta(x - a)]$$

- (a) Write down the wavefunctions for  $x < 0$ ,  $0 < x < a$  and  $x > a$  for potential bound states, with justification.
- (b) What must be the boundary conditions at  $x = 0$  and  $x = a$ ?
- (c) Take mass  $m$  to be the free electron mass,  $V_0 = 10\text{eV}$  and  $L = 1\text{\AA}$ . Calculate the minimum distance  $a$  for an antisymmetric wavefunction to exist. Your answer must be a number, expressed in angstrom.
21. A compensated (containing both donor and acceptor impurity atoms in the same region) n-type Si sample at  $T = 300\text{ K}$ , shows a conductivity of  $\sigma = 16\ \Omega\text{cm}^{-1}$  and an acceptor doping concentration of  $10^{17}\ \text{cm}^{-3}$ . Assume complete ionization and an electron mobility of  $\mu_n = 1500\text{cm}^2\text{V}^{-1}\text{s}^{-1}$ . The intrinsic carrier concentration is in Si at 300K is  $n_i = 1 \times 10^{10}\text{cm}^{-3}$ .
- (a) Determine the donor concentration.
- (b) Where is the Fermi level w.r.t the bottom of the conduction band?
- (c) You have not been given the hole mobility  $\mu_p$ . Can you explain *using your calculations* why it is not *necessary* to know that here?
22. Indium Antimonide has a band gap  $E_g = 0.23\text{eV}$ , relative dielectric constant  $\epsilon_r = 18$ , electron effective mass  $m^*/m_0 = 0.015$ .
- (a) Calculate the donor ionisation energy (in eV) and the radius of the ground state orbit.
- (b) At what minimum donor concentration will appreciable overlap effects between orbits of neighbouring impurity atoms occur? *Give a numerical answer - not an algebraic expression.* This approximately marks the point when the dopant level no longer remains a single level but starts spreading out into a band.
23. Consider a Silicon sample with a carrier density  $n = 10^{17}\text{cm}^{-3}$  and electron mobility  $\mu_n = 700\text{cm}^2\text{V}^{-1}\text{s}^{-1}$ . The electron effective mass may be taken to be  $0.2m_0$  where  $m_0$  is the free electron mass.

- (a) Calculate the classical thermal velocity ( $v_{th}$ ) and the Fermi velocity ( $v_F$ ).
- (b) Calculate the ratio  $\frac{v_F}{v_{th}}$ . For a good metal (*e.g.* Cu, Ag, Au, Al *etc.*) how do you expect this ratio to change?
- (c) Estimate the electric field (in Volts/cm) at which the carrier drift velocity will reach  $\sim 10\%$  of the thermal velocity.
24. Consider a  $p^+n$  junction in reverse bias. The notation means that the p side is much more heavily doped ( $N_a \gg N_d$ ). At 30 V reverse bias, the junction capacitance is measured to be  $1.7\text{nF}\cdot\text{cm}^{-2}$ . The material of the junction is known to breakdown around an electric field  $E = 3 \times 10^5 \text{V}\cdot\text{cm}^{-1}$  and its relative dielectric constant  $\epsilon_r = 12$ . Estimate the voltage at which this junction would break down.
25. If the intrinsic carrier density is comparable to the carrier density contributed by the ionised dopants, then the distinction between n and p type material also breaks down. Then a p-n junction would obviously no longer function as a rectifier or a diode. Using this concept estimate the maximum working temperature of a Si diode and a GaN diode. You are given that
- The band gaps for Si, GaN are 1.1 and 3.4 eV,
  - the intrinsic carrier concentrations at 300K are  $10^{10}\text{cm}^{-3}$  and  $10^{-9}\text{cm}^{-3}$ .
  - The doping level is  $10^{15}\text{cm}^{-3}$  for all.
26. The barrier voltage  $V_0$  of a p-n junction is usually written in the form

$$V_0 = \frac{kT}{|e|} \ln \frac{N_a N_d}{n_i^2}$$

where  $N_a$  and  $N_d$  are the doping concentrations on the p and n side respectively.  $n_i$  is the intrinsic carrier density.

- (a) Do you expect this quantity to increase or decrease with temperature? Answer with a simple calculation.
- (b) Suppose  $N_a = N_d = 5 \times 10^{17}\text{cm}^{-3}$ , the effective masses are 0.9 and 0.2  $m_0$  respectively.  
Estimate  $\frac{\delta V_0}{\delta T}$  at  $T = 300\text{K}$
27. Consider a two dimensional material with a square lattice with lattice constant  $a$ . The conduction band dispersion relation is given to be

$$E(\mathbf{k}) = E_0 \left( \sin^2 \frac{k_x a}{2} + \sin^2 \frac{k_y a}{2} \right)$$

This is of course the tight binding result, you can take it as given, where  $E_0$  is a constant.

- (a) What would be the shape of the Fermi surface for very low carrier densities? Answer with a simple figure and calculation.
- (b) If the Fermi energy  $E_F = E_0$ , what would be the shape of the Fermi surface? Calculate and sketch the shape. What fraction of the first Brillouin zone is filled if  $E_F = E_0$  ?
- (c) What simple feature distinguishes the Fermi surfaces for  $E_F < E_0$  and  $E_F > E_0$ ?
- (d) If the lattice constant is made smaller do you expect  $E_0$  to increase or decrease? Answer with reason.
- (e) Now consider the 3 dimensional simple cubic lattice where the dispersion is

$$E(\mathbf{k}) = E_0 \left( \sin^2 \frac{k_x a}{2} + \sin^2 \frac{k_y a}{2} + \sin^2 \frac{k_z a}{2} \right)$$

Assume that the bandgap is very large and you are asked to adjust the doping so that  $E_F = E_0/20$ . What is the required minimum donor concentration?

- (f) The effective Bohr radius of the material is  $a_B = 5\text{nm}$  and  $a = 5\text{\AA}$ . At this doping level do you expect the semiconductor to be metallic or insulating?
28. Consider two electron bands with populations  $n_1$  and  $n_2$  (per unit area) and mobilities  $\mu_1$  and  $\mu_2$  existing in a semiconductor. You are given a thin wafer of the material in the shape of a rectangle, placed in the  $xy$  plane. A magnetic field is applied perpendicular to it  $\mathbf{B} = (0, 0, B)$  and the Hall voltage is measured by sending a current in the  $x$  direction and measuring the voltage in the  $y$  direction.
- (a) Write down the basic matrix equations connecting the current and electric fields.

(b) Show that the measured Hall coefficient (in the low field regime) is

$$R_H = \frac{E_y}{j_x} = -\frac{1}{e} \frac{n_1\mu_1^2 + n_2\mu_2^2}{(n_1\mu_1 + n_2\mu_2)^2} B$$

(c) State clearly what "low field" means in this context.

(d) Is the slope of  $R_H$  vs  $B$  larger than  $\frac{1}{(n_1 + n_2)e}$  or smaller?

29. Consider a semiconductor with an energy band dispersion given by

$$\mathcal{E}(k_x, k_y) = A(k_x^2 + 2k_y^2) - B(3k_x^4 + k_y^4)$$

where  $A, B$  are positive constants.

(a) Now if an electric field is applied in the (1,1) direction, in which direction would an electron at  $(k_x = 0, k_y = 0)$  accelerate? Your answer must be an angle only (e.g. making  $20^\circ$  to  $k_x$  axis).

(b) Find at least one point (or points) around which you expect a "hole" like behaviour.

30. The conduction band minima of Silicon occurs at 6 equivalent regions as shown in the figure.

For the valley in the [100] direction the dispersion near the minima can be written as

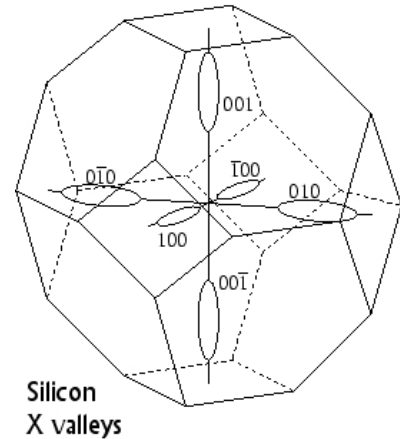
$$\mathcal{E}(\mathbf{k}) = \mathcal{E}_c + \frac{\hbar^2}{2m_0} \left[ \frac{(k_x - k_0)^2}{m_L} + \frac{k_y^2}{m_T} + \frac{k_z^2}{m_T} \right]$$

where  $m_L = 0.98$  and  $m_T = 0.19$  and  $m_0$  is the free electron mass.

Given this information, calculate the conduction band density of states,  $N_C$  at  $T = 300\text{K}$ , including all degeneracies, where

$$N_C = \int_{\mathcal{E}_c}^{\infty} D(\mathcal{E}) \exp\left(-\frac{\mathcal{E} - \mathcal{E}_c}{kT}\right) d\mathcal{E}$$

where  $D(\mathcal{E})$  is the density of states.



31. Two samples of GaAs are doped with  $10^{17}\text{cm}^{-3}$  and  $10^{18}\text{cm}^{-3}$  of donors respectively. The shallow donor level is 6 meV below the conduction band.

(a) Calculate the location of the Fermi level (w.r.t. the conduction band bottom) and the fraction of donors that will be ionised at room temperature ( $T=300\text{K}$ ) for each sample.

(b) Under what condition will almost all the dopants ionise at room temperature?

The conduction band effective density of states for GaAs is  $N_C = 4.7 \times 10^{17}\text{cm}^{-3}$  at  $T = 300\text{K}$

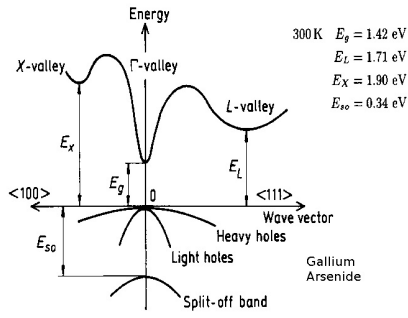
32. You are given a TWO dimensional semiconductor with parabolic valence and conduction bands, where the effective masses are  $m_h$  and  $m_e$  respectively. There are no dopants put in.

(a) Find the location of the Fermi level as a function of temperature. Your answer should be an algebraic relation involving  $E_c$ ,  $E_v$  and the effective masses only.

(b) In what way does your result differ from the three dimensional case?

(c) In what (hypothetical) situation would the intrinsic (no doping condition) Fermi level be independent of temperature?

33. The conduction band of a material has two minimas as shown schematically in the figure (marked  $\Gamma$  and  $L$ .)



The first minima is at  $\Gamma$ -point,  $\mathbf{k} = (0, 0, 0)$ , suppose the second ( $L$ ) minima is at  $\mathbf{k} = (k_0, 0, 0)$ . In reality it is in the (1,1,1) direction, but that does not matter for what you are going to do. Initially only a few states in the  $\Gamma$  "valley" are populated. An electric field  $F$  is then applied along  $k_x$ . Estimate the *minimum* mobility at which a significant number of carriers may end up in the other valley. Make any assumptions you want but clearly state what you are assuming! (Your answer must be a simple algebraic expression, no implicit integrals etc....) Such a phenomena actually happens in Gallium Arsenide, which is shown - but you do not need to know any of the specific details of it to make the estimate.

34. Two ends of a uniformly doped bar of an N-type semiconductor are illuminated with light. The two ends are at  $x = -L$  and  $x = L$ , no light falls/penetrates in the region  $-L < x < L$ . The doping concentration is  $N_d \gg n_i$  and the light creates an excess carrier density of  $\gamma N_d$  (where  $\gamma \ll 1$ ) at the ends.

Find the variation of the hole concentration  $p(x)$  along the bar. What is hole concentration at  $x = 0$ ?

You can assume usual symbols for minority carrier lifetimes, diffusivities, diffusion lengths (*i.e.*  $\tau_p$ ,  $D_p$ ,  $L_p$ ) etc.