The wave equation, Lorenz force law and classical mechanics

Light	Sound
$c = \sqrt{\frac{1}{\epsilon_0 \mu_0}}$	$v = \sqrt{\gamma \frac{P}{\rho}}$
Can propagate in vacuum. So the velocity is w.r.t. what?	Needs a medium. The velocity is w.r.t the medium (like air, water)

The ether frame is a hypothetical inertial frame in which the speed of light was supposed to be c

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Velocity may be different in different intertial frames

So how is the force going to be same in all intertial frames?

Maxwell's equations are NOT invariant under a Gallilean transformation

What does "Laws of physics are same in all intertial frames" mean?

Suppose Z = X * Y is a law of physics.

In an intertial frame (S) some one measures the quantitites to be X, Y and Z

In another intertial frame (S') one measures the quantitites to be X', Y' and Z'

S will find Z = X * Y

S' will find Z' = X' * Y'

Further S will be able to predict what X', Y', Z' will be measured in the other frame.

This is the job of the transformation equations.

In general X,Y,Z and X',Y', Z' will not be same.

What if time (or time interval) is one of the quantities ? For example half life of a radioactive substance.

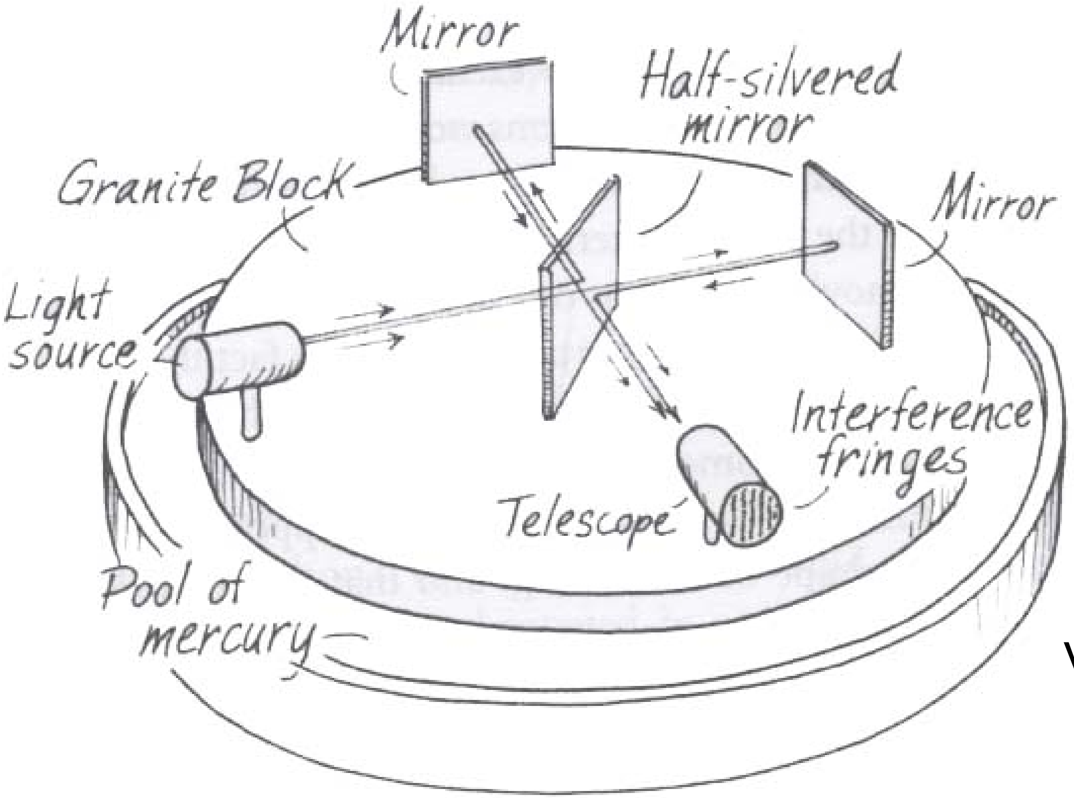
Time was thought to be universal....but that is NOT unambiguous.

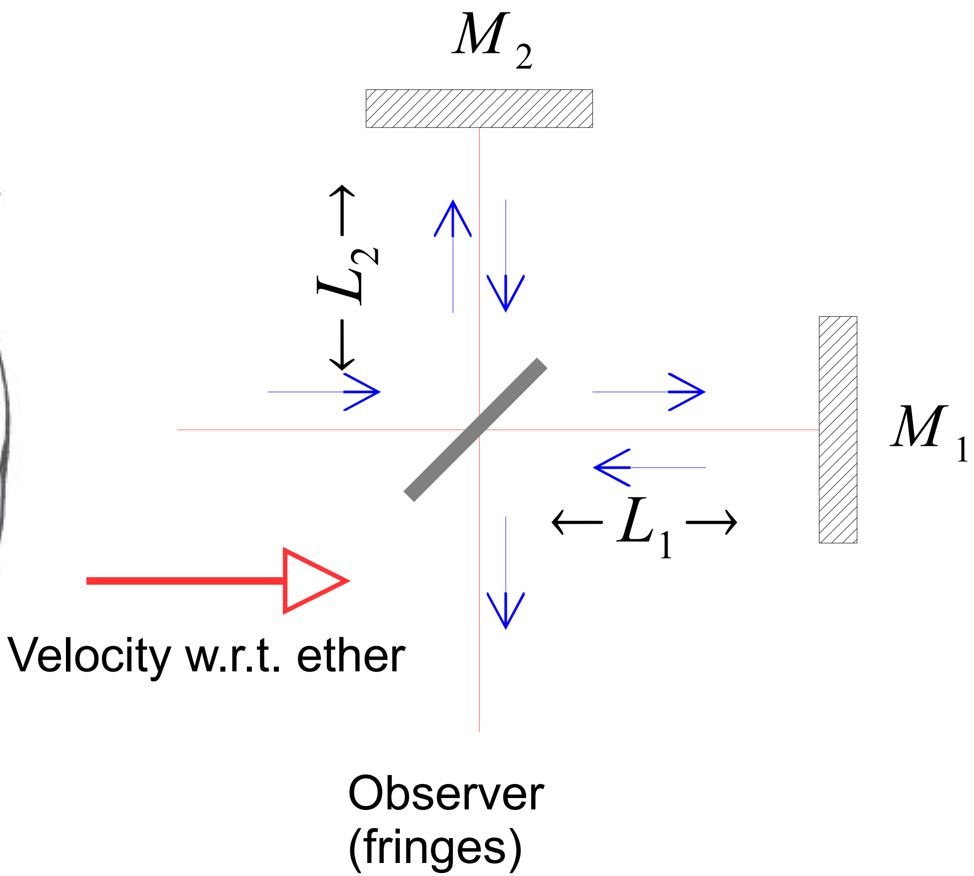
Newton : The universe HAS a clock.

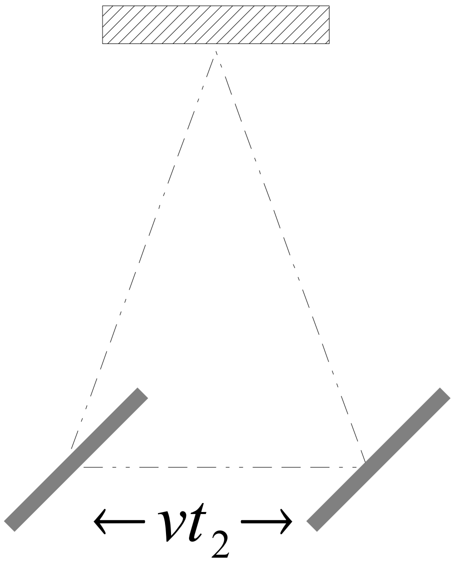
Liebnitz : The universe IS a clock.

What is the difference ?

Michaelson Morley interferometer experiment







The reflection from M2
as seen by an observer
in the ether frame

Michaelson Morley interferometer experiment

To reach M_1 light takes : $\dfrac{L_1}{c-v}$

To come back : $\dfrac{L_1}{c-v}$

$$t_1 = \frac{2L_1 c}{c^2-v^2}$$

To reach M_2 and come back : $L_2^{\,2}+\left(\dfrac{vt_2}{2}\right)^2 = \left(\dfrac{ct_2}{2}\right)^2$

$$\Delta t = t_1 - t_2 = \frac{2L_1 c}{c^2-v^2} - \frac{2L_2}{\sqrt{c^2-v^2}}$$

Now turn the whole apparatus by 90°

$$\Delta t' = t_1' - t_2' = \frac{2L_1}{\sqrt{c^2-v^2}} - \frac{2L_2 c}{c^2-v^2}$$

$$\Delta t - \Delta t' = \frac{2(L_1+L_2)c}{c^2-v^2} - \frac{2(L_1+L_2)}{\sqrt{c^2-v^2}} \approx \left(\frac{L_1+L_2}{c}\right)\frac{v^2}{c^2}$$

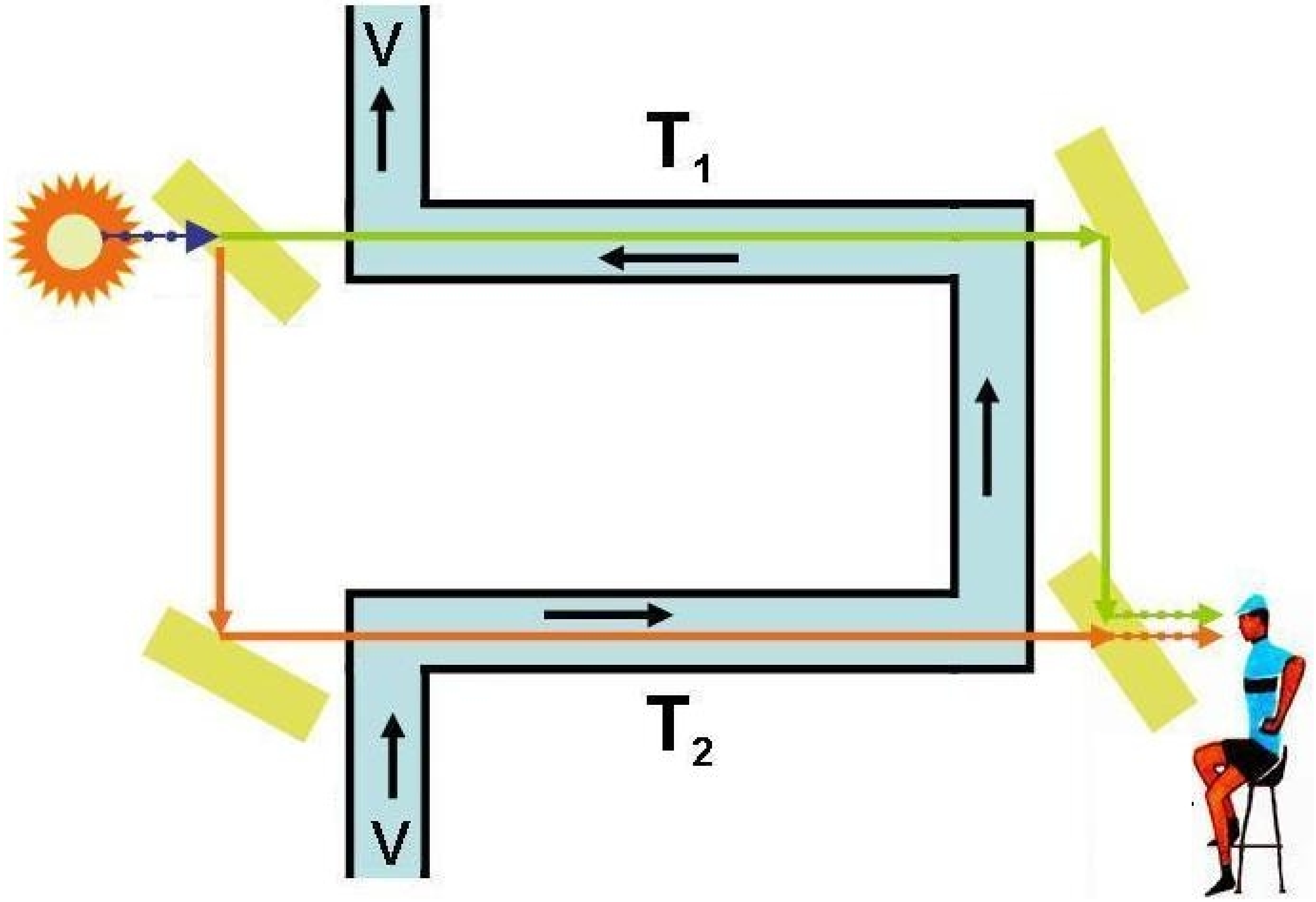
path diff λ
one fringe shift

In experiment : $L_1+L_2 \approx 22\,mt$: $\lambda = 500\text{nm}$

sensitivity 1/100 fringe : expect ~ 0.4 fringe shift for $\dfrac{v}{c} \approx 10^{-4}$

NO SHIFT was observed

Light in a moving medium : Fizeau



Fringe pattern changes when the flow is stopped

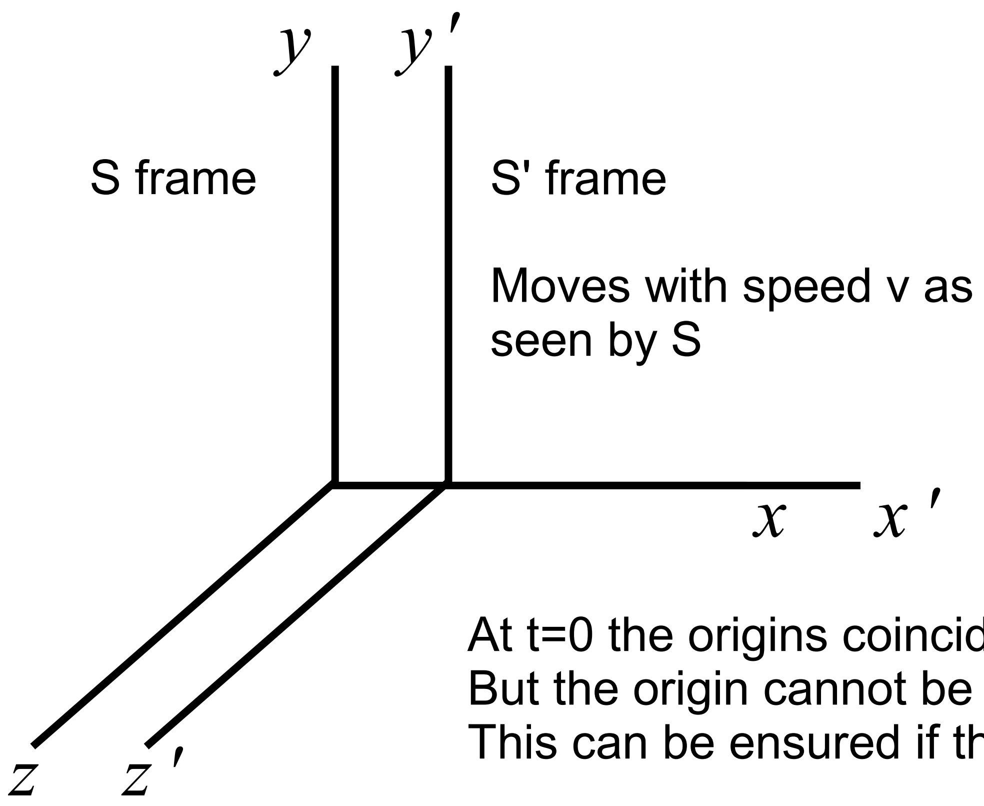
Infer the velocity of light from the fringe shift data

$$v_{light} = \frac{c}{n} + v_{water}\left(1 - \frac{1}{n^2}\right)$$

Light appears to be partially "dragged" by the flowing medium.

The postulates of special relativity & Lorenz transformation equations

1. The laws of physics are same in all intertial frames.
2. The speed of light in vacuum (c) is same in all inertial frames.



At t=0 the origins coincide.
But the origin cannot be a special point.
This can be ensured if the transformation is LINEAR

The postulates of special relativity & Lorenz transformation equations

$$
\begin{aligned}
x' &= a_{11}x + \cancel{a_{12}}y + \cancel{a_{13}}z + a_{14}t \\
y' &= \cancel{a_{21}}x + a_{22}y + \cancel{a_{23}}z + \cancel{a_{24}}t \\
z' &= \cancel{a_{31}}x + \cancel{a_{32}}y + a_{33}z + \cancel{a_{34}}t \\
t' &= a_{41}x + \cancel{a_{42}}y + \cancel{a_{43}}z + a_{44}t
\end{aligned}
$$

A point on the x-z plane goes over to a point on the x'z' plane

A point on the x-y plane goes over to a point on the x'y' plane

Clocks placed symmetrically below and above the x'z' plane should not give different time. Up and down can not be different!

Clocks placed symmetrically below and above the x'y' plane should not give different time. Up and down can not be different!

x=vt and x'=0 must coincide at all times

The postulates of special relativity & Lorenz transformation equations

A rod of length L stands along the y axis in S frame.

S' measures the length to be a_{22}

Bring the rod to rest in S'.

S measures the length to be $1/a_{22}$

equivalence of reference frames requires that $a_{22} = \frac{1}{a_{22}} = 1$

Same argument holds for $a_{33} = 1$

The equations reduce to

$$
\begin{aligned}
x' &= a_{11}(x - vt) \\
y' &= y \\
z' &= z \\
t' &= a_{41}x + a_{44}t
\end{aligned}
$$

Three unknowns, so we need three equations.

The postulates of special relativity & Lorenz transformation equations

1. Consider a light pulse sent along +x at time t=0 by S
   At time t, the pulse has co-ordinates (ct, t)

$$x' = a_{11}(ct - vt)$$
$$t' = a_{41}ct + a_{44}t$$

$$\text{But } \frac{x'}{t'} = c \quad \Rightarrow \quad \boxed{\frac{a_{11}}{a_{41}c + a_{44}} = \frac{c}{c - v}}$$

2. Consider a light pulse sent along -x at time t=0 by S
   At time t, the pulse has co-ordinates (-ct, t)

$$x' = a_{11}(-ct - vt)$$
$$t' = -a_{41}ct + a_{44}t$$

$$\text{But } \frac{x'}{t'} = -c \quad \Rightarrow \quad \boxed{\frac{a_{11}}{-a_{41}c + a_{44}} = \frac{c}{c + v}}$$

3. Consider a light pulse sent along +y by S
At time t the pulse has co-ordinates (0,ct,0,t)

$$
\begin{aligned}
x' &= a_{11}(0 - vt) \\
y' &= ct \\
z' &= 0 \\
t' &= a_{41}.0 + a_{44}t
\end{aligned}
$$

Speed measured by S'

$$\frac{\sqrt{a_{11}^{2}v^{2}+c^{2}}}{a_{44}} = c$$

Solving for the 3 unknowns

$$a_{11} = a_{44} = \frac{1}{\sqrt{1-v^2/c^2}}$$

$$a_{41} = -\frac{v/c^2}{\sqrt{1-v^2/c^2}}$$

$$
\begin{aligned}
x' &= \frac{x-vt}{\sqrt{1-v^2/c^2}} \\
y' &= y \\
z' &= z \\
t' &= \frac{t-vx/c^2}{\sqrt{1-v^2/c^2}}
\end{aligned}
$$

Invert these equations. What result should you expect?
Do you see that happening?

How does one measure length? Why is time involved in this meansurement?

Measure the co-ordinates of two ends of a stick at the same instant of time.

Is "at the same instant" (simultaneous) an unambiguous notion?

In an inertial frame, yes . But NOT across different frames.

Two clocks may be synchronised in frame S. To S' they will not be so.

A rod of length L is at rest in S' : between x_2 and $x_1{}'$

S' measures the rest frame length $L' = x_2{}' - x_1{}'$

S tries to measure the length

he must measure the co-incience of the endpoints at the same instant.

$$\left.\begin{aligned} x_1{}' &= \frac{x_1 - vt}{\sqrt{1-\beta^2}} \\ x_2{}' &= \frac{x_2 - vt}{\sqrt{1-\beta^2}} \end{aligned}\right\} \quad L = (x_2{}' - x_1{}')\sqrt{1-\beta^2} = L'\sqrt{1-\beta^2}$$

How does one compare measurement of time intervals?

Consider a clock at rest in the S' frame at the origin of S' (so x'=0 always)

At t'=0 we have t=0.

What happens when t' =1 (say)

A clock at rest in S' will appear to run slow when timed by clocks in S.

$$t = \frac{t' + vx'/c^2}{\sqrt{1-\beta^2}}$$

$$x' = 0 \quad \text{and} \quad t' = 1$$

$$t = \frac{1}{\sqrt{1-\beta^2}}$$

If the "clock" is a particle with a 1 second lifetime it will appear to live longer when seen by S.

This is why many fast moving particles in cosmic rays manage to reach the earth's surface.

"Length contraction" and "time dilation" are consequences of the two postulates of relativity.

Transformation of velocities : '' velocity addition rule“

Problem: We need to relate $u_x = \frac{dx}{dt}$ with $u_x' = \frac{dx'}{dt'}$

$$\delta x' = \frac{\delta x - v\,\delta t}{\sqrt{1-\beta^2}}$$

$$\delta t' = \frac{\delta t - (v/c^2)\,\delta x}{\sqrt{1-\beta^2}}$$

$$\frac{\delta x'}{\delta t'} = \frac{\delta x - v\,\delta t}{\delta t - (v/c^2)\,\delta x}$$

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2}$$

$$u_x = \frac{u_x' + v}{1 + u_x' v/c^2}$$

what happens if

$u_x' = \frac{c}{n}$? (Fizeau)

The acceleration seen by two inertial observers is not the same.
Try to derive the relation by calculating the second derivatives and relating them.

How would you define Force ?

The invariant quantites and "four vectors"

x^2	+	y^2	+	z^2	-	$c^2 t^2$	position-time
$p_x{}^2$	+	$p_y{}^2$	+	$p_z{}^2$	-	E^2/c^2	energy-momentum
$j_x{}^2$	+	$j_y{}^2$	+	$j_z{}^2$	-	$\rho^2 c^2$	current-density-charge
$A_x{}^2$	+	$A_y{}^2$	+	$A_z{}^2$	-	ϕ^2/c^2	four-potential

Space like   **Time like**

These combinations remain invariant under a Lorenz transformation.

Like the magnitude of a vector remains fixed under rotation of co-ordinates

These combinations are called Four vectors

They transform in exactly the same way (x,y,z, t) does

The Electric and magnetic fields do not transform in a simple way!

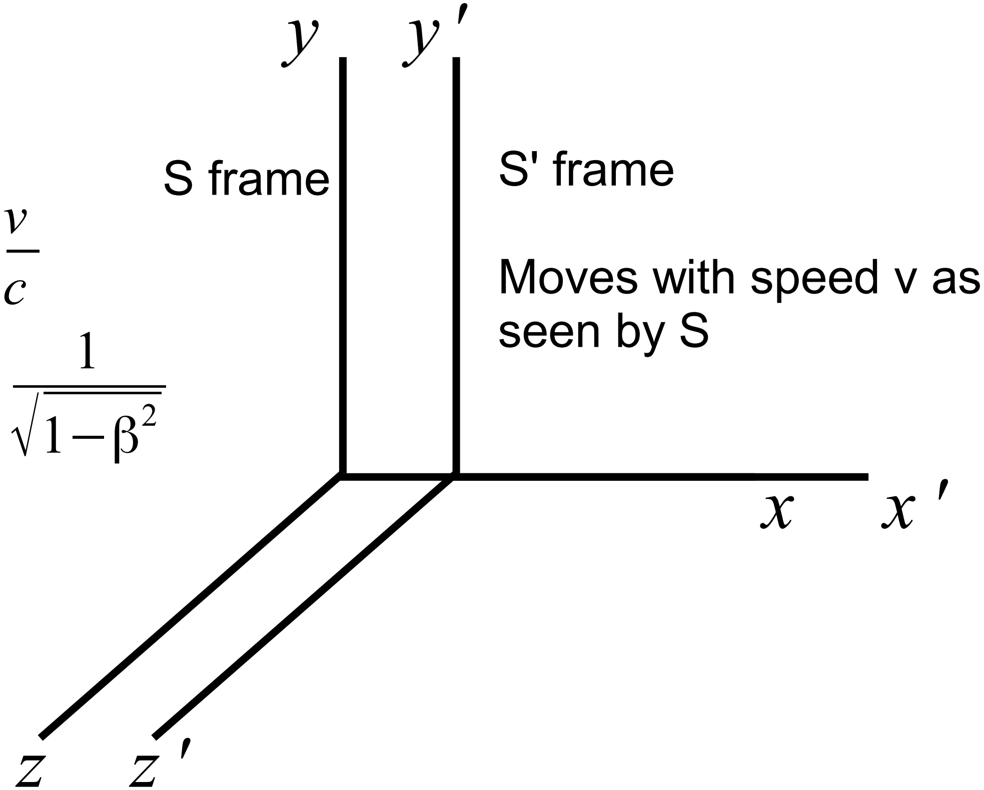
The transformation of electric and magnetic fields

$$
\begin{aligned}
E_x' &= E_x \\
E_y' &= \gamma\left(E_y - vB_z\right) \\
E_z' &= \gamma\left(E_z + vB_y\right)
\end{aligned}
$$

$$
\begin{aligned}
B_x' &= B_x \\
B_y' &= \gamma\left(B_y + \frac{v}{c^2}E_z\right) \\
B_z' &= \gamma\left(B_z - \frac{v}{c^2}E_y\right)
\end{aligned}
$$

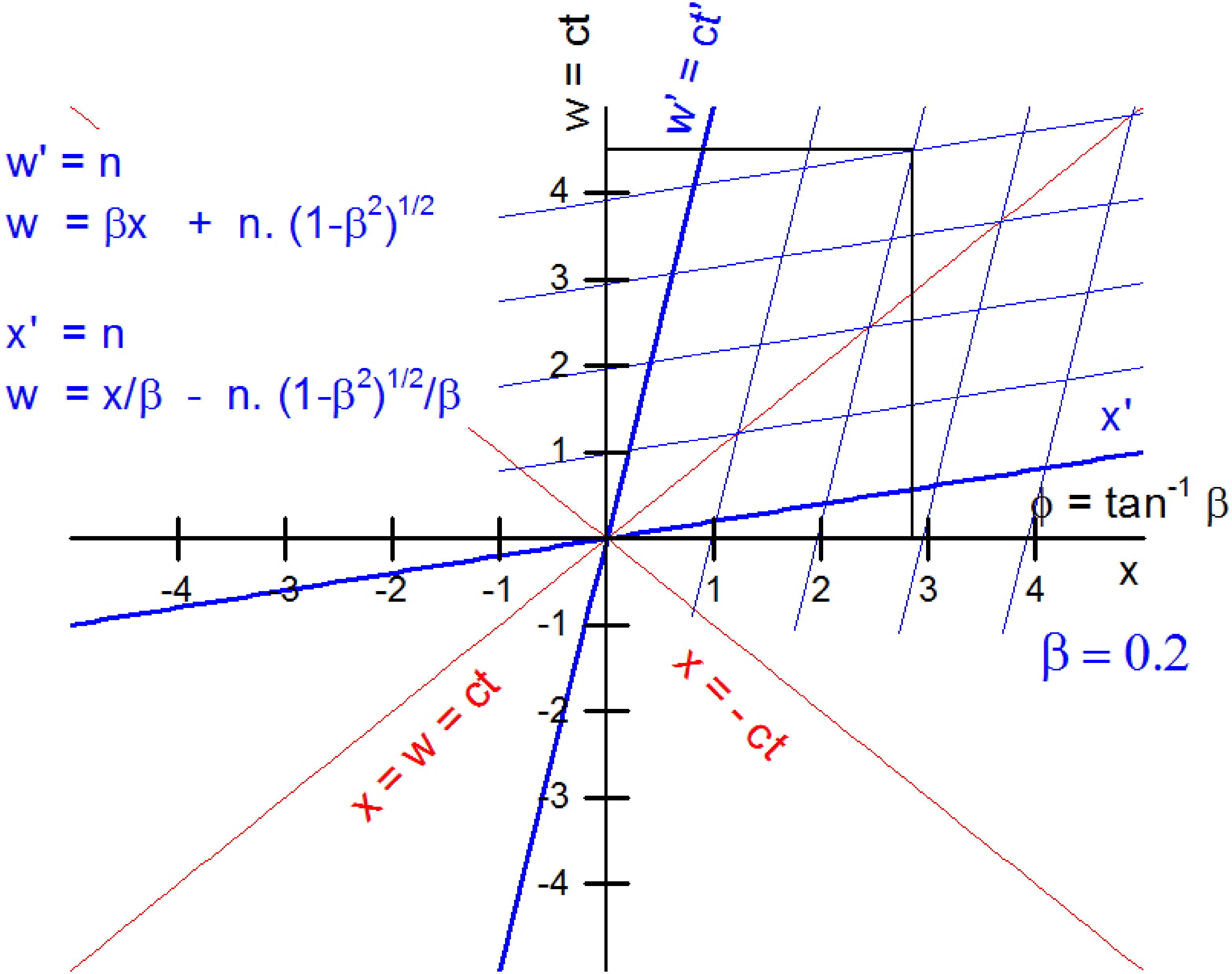
$$
\beta = \frac{v}{c}
$$

$$
\gamma = \frac{1}{\sqrt{1-\beta^2}}
$$

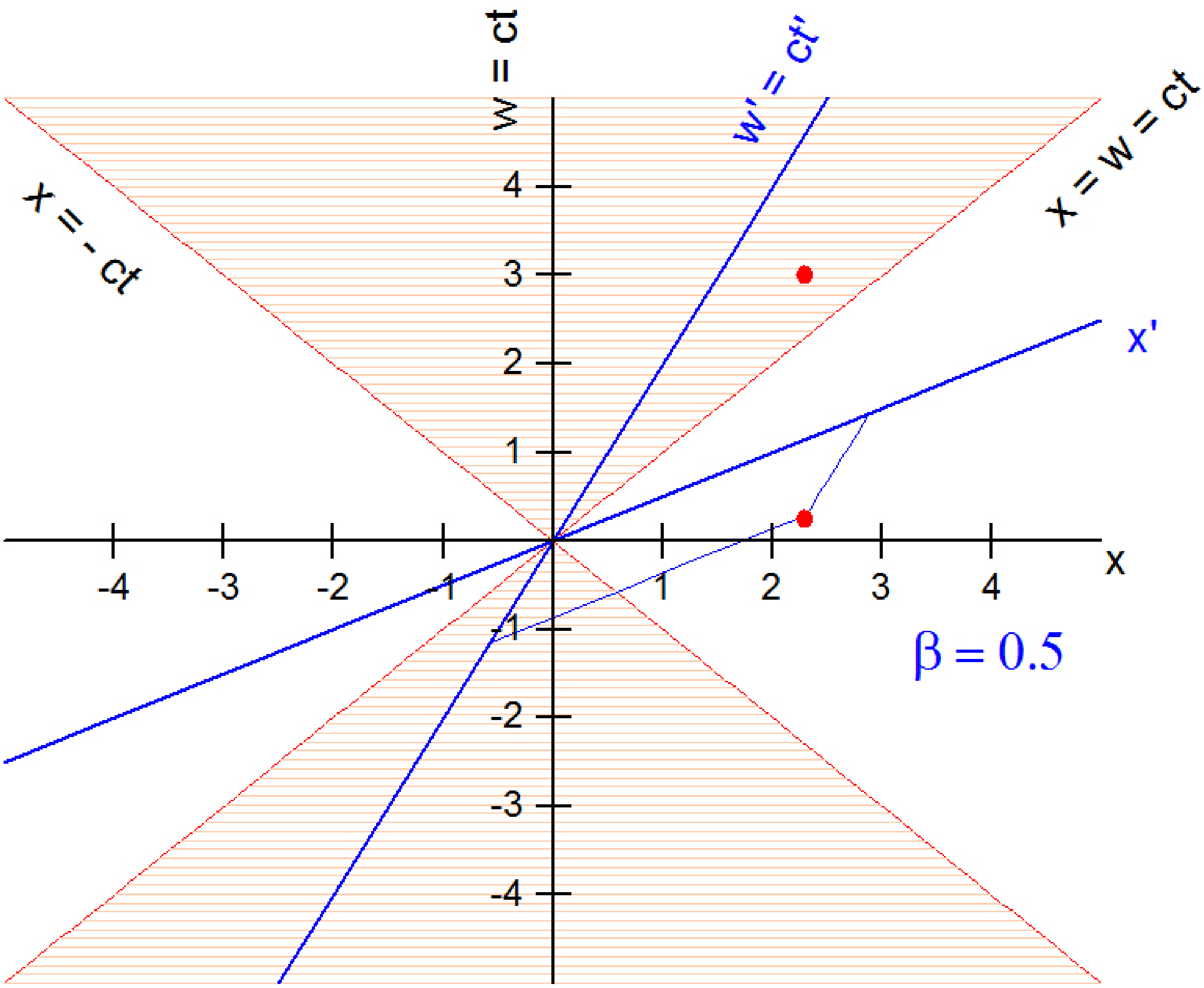


Problem: Consider a point charge +Q at rest in the origin of the S' frame.
What is the field as seen by S' ?
Use the transformation equations to derive E and B of a point charge in motion.

A geometric represtation of Lorenz transformation

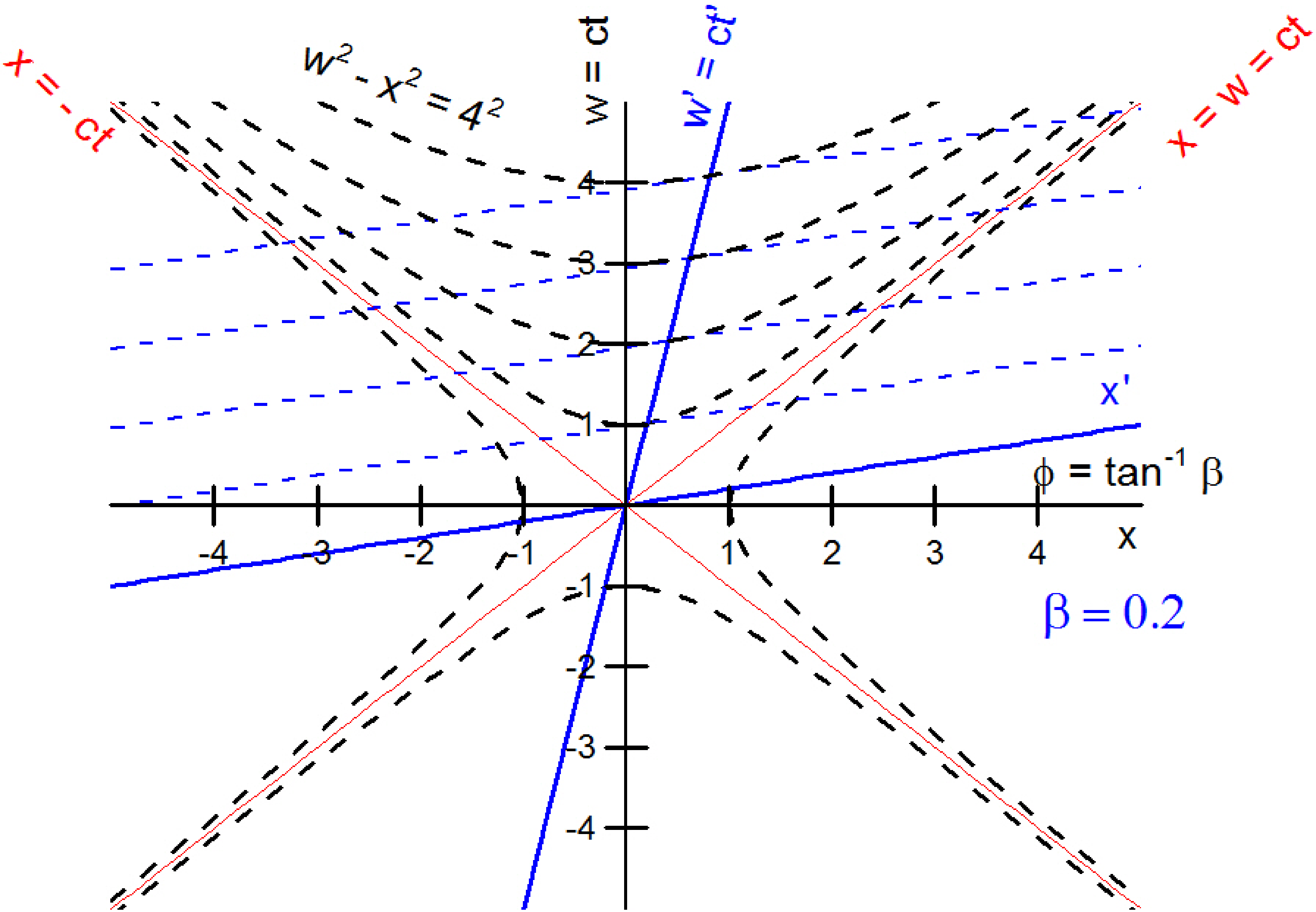


A geometric represtation of Lorenz transformation (past and present)



The time ordering of some events will be reversed. Why is that not a problem?

A geometric represtation of Lorenz transformation



A geometric represtation of Lorenz transformation