Frequency Response of Feedback Amplifiers

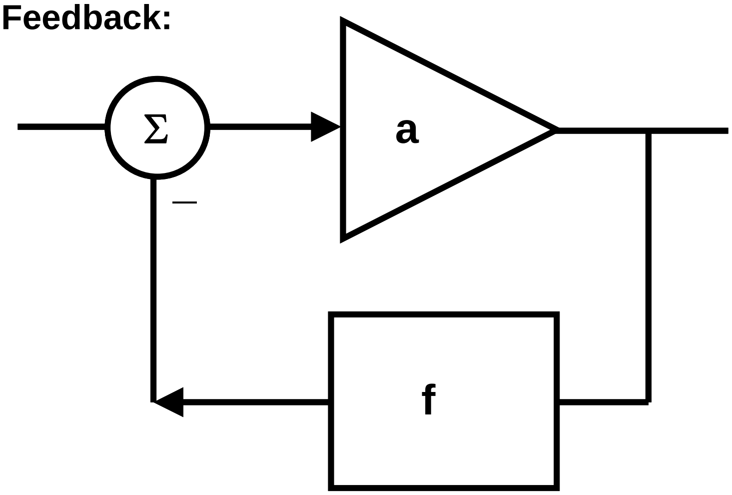
Reading:

1. Gray and Meyer, *Analysis and Design of Analog Integrated Circuits*, Third Ed., J. Wiley, 1993. Sect. 9.1 – 9.4, and Sect. 9.5.4.

2. T. H. Lee, *The Design of CMOS Radio-Frequency Integrated Circuits*, Cambridge Univ. Press, 1998. Chap. 14.

Feedback Amplifiers: One and Two Pole cases

Negative Feedback:



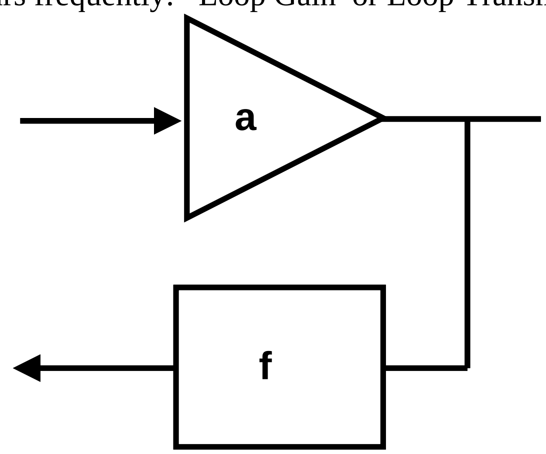
There must be 180° phase shift somewhere in the loop. This is often provided by an inverting amplifier or by use of a differential amplifier.

Closed Loop Gain: $A = \frac{a}{1 + af}$

When a >> 1, then $A \cong \frac{a}{af} = \frac{1}{f}$

This is a very useful approximation.

The product af occurs frequently: Loop Gain or Loop Transmission T = af



At low frequencies, the amplifier does not produce any excess phase shift. The feedback block is a passive network.

But, all amplifiers contain poles. Beyond some frequency there will be excess phase shift and this will affect the stability of the closed loop system.

Frequency Response

Using negative feedback, we have chosen to exchange gain a for improved performance

Since A = 1/f, there is little variation of closed loop gain with a.
Gain is determined by the passive network f

But as frequency increases, we run the possibility of

- Instability
- Gain peaking
- Ringing and overshoot in the transient response

We will develop methods for evaluation and compensation of these problems.

Bandwidth of feedback amplifiers: Single Pole case

Assume the amplifier has a frequency dependent transfer function

$$A(s)=\frac{a(s)}{1+a(s)f}=\frac{a(s)}{1+T(s)}$$

$$and$$

$$a(s)=\frac{a_o}{1-s/p_1}$$

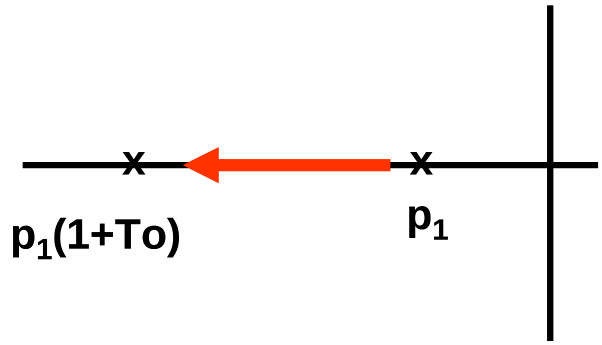
where p_1 is on the negative real axis of the s plane.

With substitution, it can be shown that:

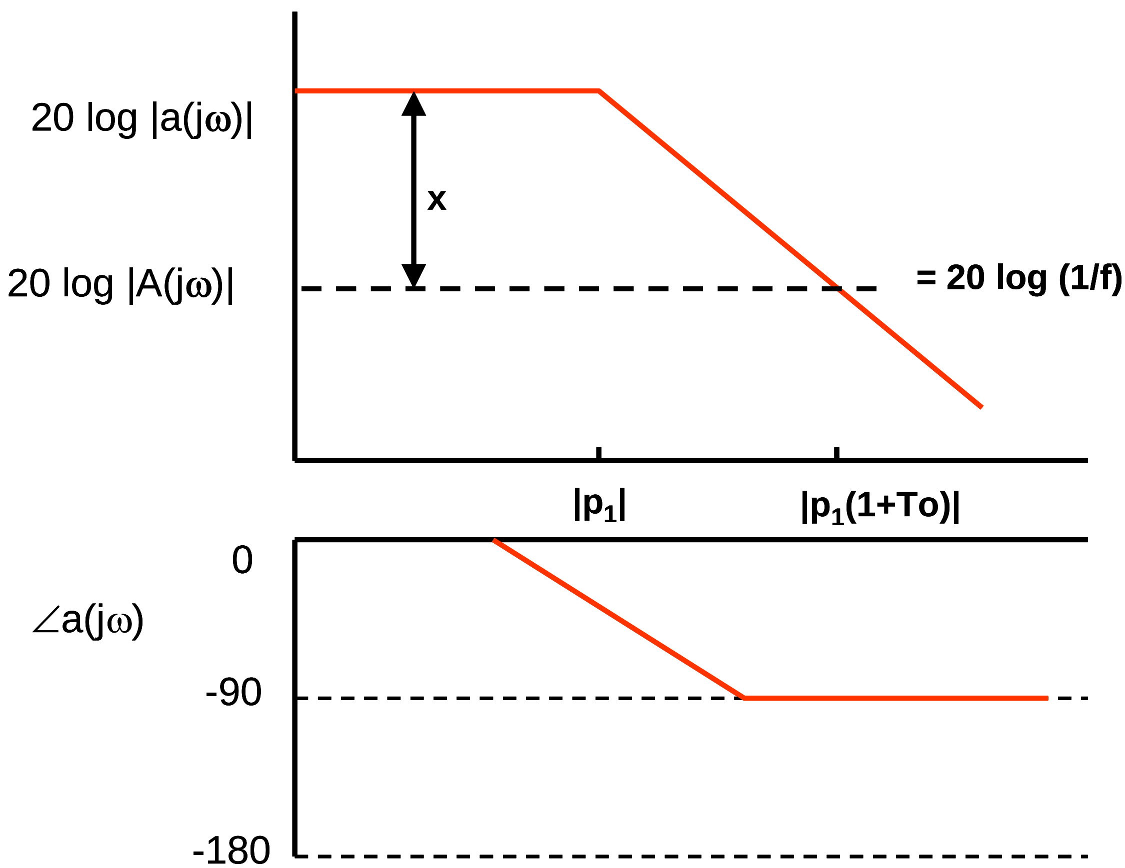
$$A(s) = \left(\frac{a_o}{1 + a_o f}\right)\left(\frac{1}{1 - s/[p_1(1 + a_o f)]}\right)$$

We see the low frequency gain with feedback as the first term followed by a bandwidth term. The 3dB bandwidth has been expanded by the factor $1 + a_o f = 1 + T_o$.

S - plane



Bode Plot



Note that the separation between a and A, labeled as x,

$$x = 20\log(|a(j\omega)|) - 20\log(1/f) = 20\log(|a(j\omega)f|) = 20\log(T(j\omega))$$

Therefore, a plot of T(jω) in dB would be the equivalent of the plot above with the vertical scale shifted to show 1/f at 0 dB.

We see from the single pole case, the maximum excess phase shift that the amplifier can produce is 90 degrees.

Stability condition:

If $|T(j\omega)| > 1$ at a frequency where $\angle T(j\omega) = -180^o$,
then the amplifier is unstable.

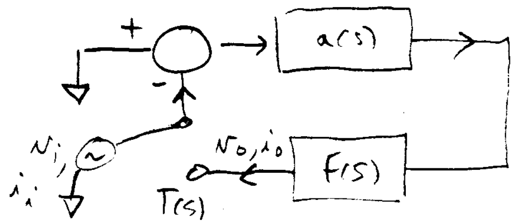
This is the opposite of the Barkhousen criteria used to judge oscillation with positive feedback. In fact, a round trip 360 degrees (180 for the inverting amplifier at low frequency plus the excess 180 due to poles) will produce positive feedback and oscillations.

This is a feedback based definition. The traditional methods using T(jω)

- Bode Plots
- Nyquist diagram
- Root – locus plots

can also be used to determine stability. I find the Bode method most useful for providing design insight. To see how this may work, first define what is meant by PHASE MARGIN in the context of feedback systems.

1. <u>Bode Plots:</u> examine loop transmission $T(s)$.



<u>open loop</u>

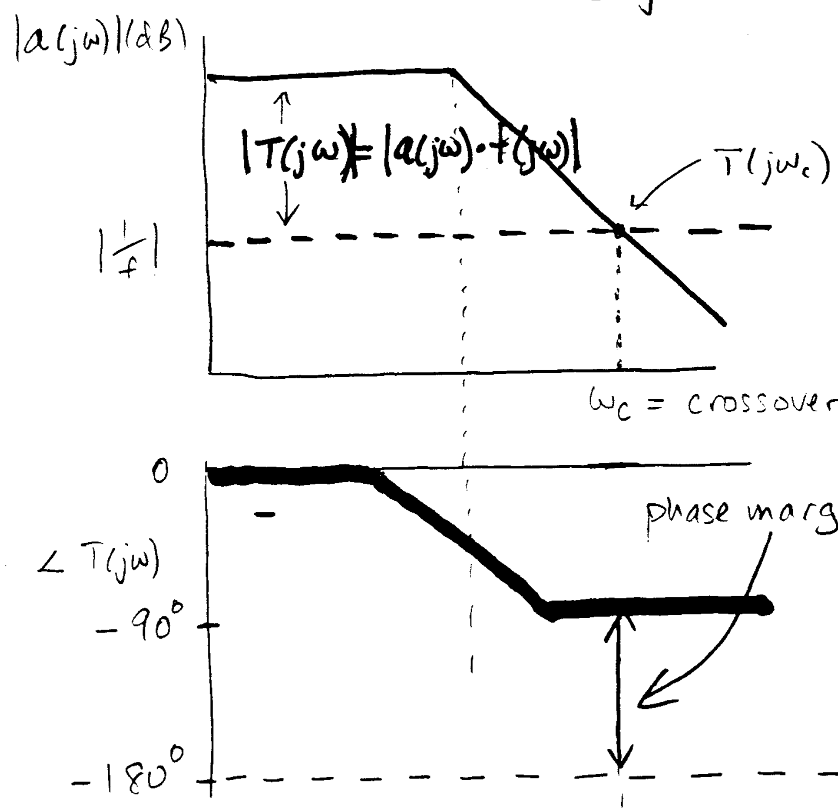
$T(s) = a(s)f(s)$

<u>BIG ADVANTAGE:</u> $T(s)$ can be

1. calculated more easily
2. simulated with spice or ADS and Bode plots generated.
3. measured on NA

<u>How does this work?</u> define gain and phase margins

$s = j\omega$

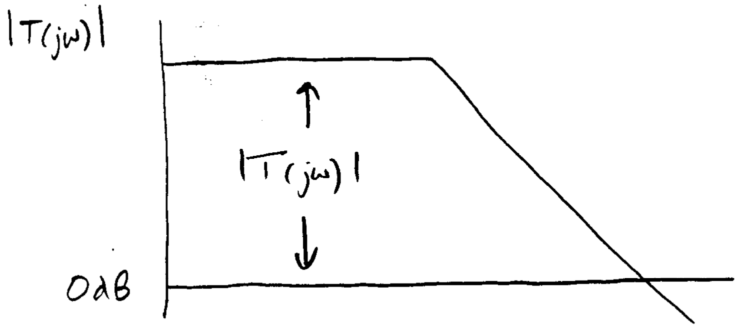


phase margin $= 180^\circ + \angle[a(j\omega_c)f(j\omega_c)]$

$= 90^\circ$

for single pole amp.

Note that: this is equivalent to plotting $|T(j\omega)|$ (dB)



← this is the $|\frac{1}{f}|$ line.

Phase margin will tell us when the loop output is getting close to being in phase - extra 180° phase shift. The larger the PM the better.

30° – 60° may be adequate in many applications.

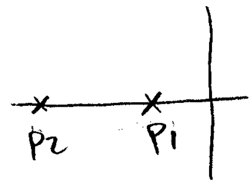
Gain margin. How much has the gain $|T(j\omega)|$ dropped below 0dB at the frequency ω_π where $\angle T(j\omega_\pi) = -180°$

for single pole case, gain margin is infinite since we never get to $-180°$.

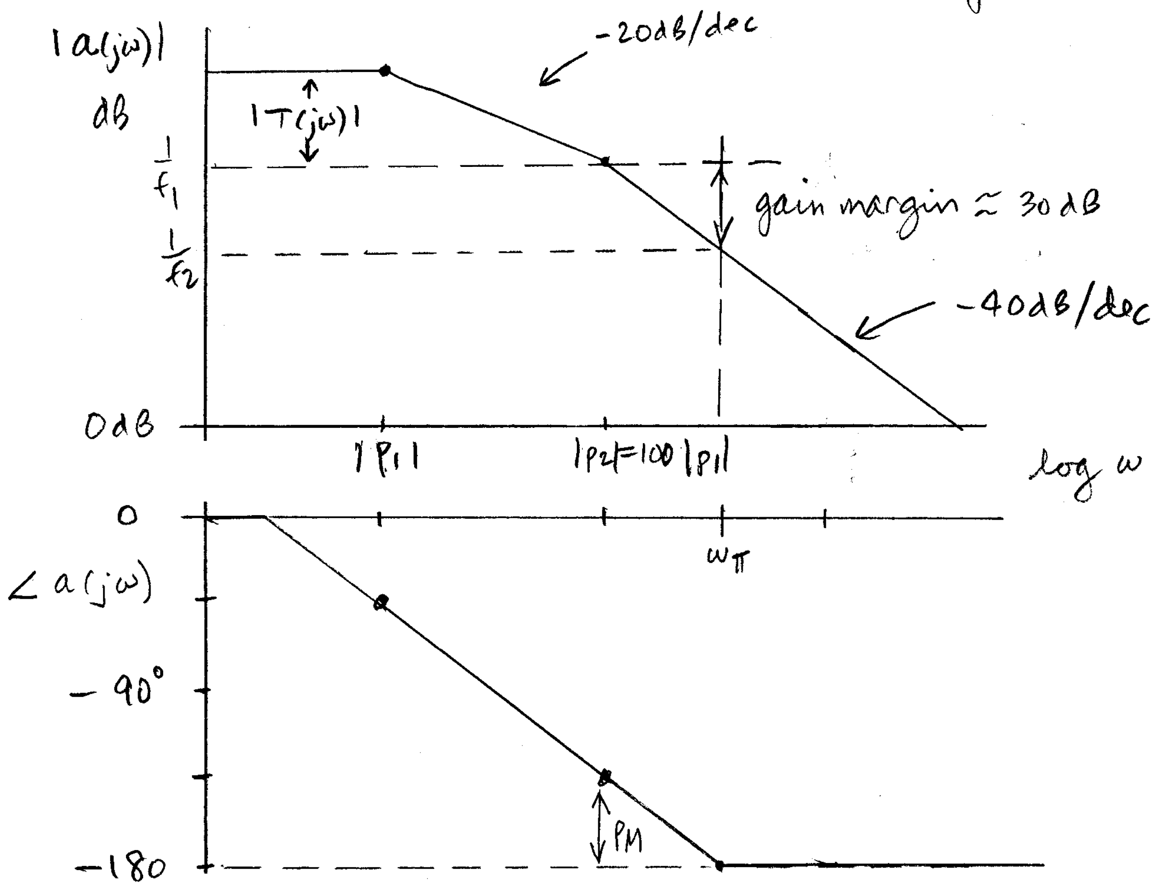
Now let's look at more complicated cases.

Second-order (two pole) system

Assume forward path has 2 poles on negative-real axis. $\underline{\underline{a(s)}}$



Draw Bode plot of $T(j\omega)$. Let's assume f is frequency independent



PM = 45° for f_1

PM = 0° @ f_2 } borderline unstable - essentially useless except for oscillator

Let's look at this another way –

Example: 2 pole FB Amplifier

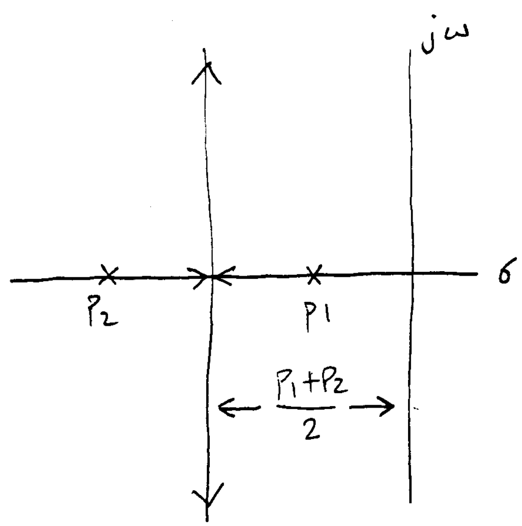
$$a(s) = \frac{a(0)}{(1 + s/p_1)(1 + s/p_2)}$$

2 real poles in LHP

we could solve for roots $1 + a(s)f = 0$

$$s^2 + s(p_1 + p_2) + (1 + a(0)f)\,p_1 p_2 = 0$$

$$s = -\frac{1}{2}(p_1 + p_2) \pm \frac{1}{2}\sqrt{(p_1 + p_2)^2 - 4(1 + a(0)f)\,p_1 p_2}$$



as f increases, poles move together, then split

large f produces widely split poles
low phase margin
ringing in transient response
peaking in frequency response

In feedback control system jargon, second order denominators are expressed as:

$$\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} s + 1$$

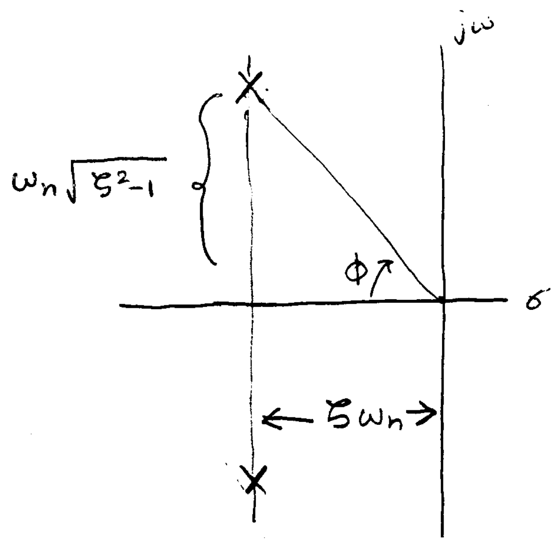
where ω_n = natural frequency

ζ = damping factor (zeta)

so, in this case,

$$\omega_n = \sqrt{(1 + a(0) f) P_1 P_2}$$

$$\zeta = \frac{P_1 + P_2}{2\omega_n}$$



$\zeta = \cos\phi$

$(0 \le \phi \le 90°)$

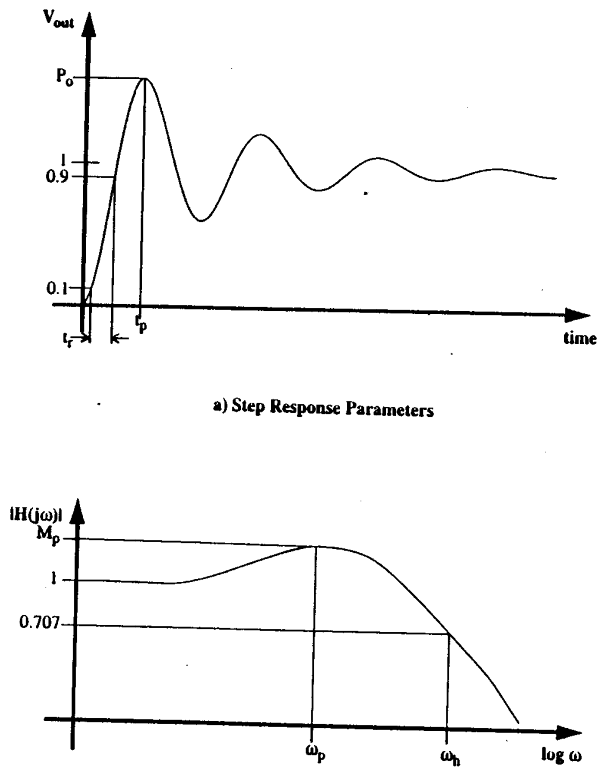
$$s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

ζ	ϕ	s	
0	$90°$	$\pm j\omega_n$	(undamped)
0.7	$45°$	$\frac{\omega_n}{\sqrt{2}} \pm j\frac{\omega_n}{\sqrt{2}}$	(flat freq. response)
1	$0°$	ω_n	(critically damped)

This notation allows use of formulas such as in Section 14.11 of Lee's text to predict bandwidth, gain peaking, overshoot, risetime, ringing.

Actually, these results, although only for a second-order feedback system, are more useful than you might at first expect. Any stable FB system will be dominated by no more than 2 poles. If there are more than this, the system must be compensated in order to be stable. (higher-order poles pushed out, cancelled, or dominant pole(s) reduced in frequency to force second-order behavior).

Cambridge, 1998.



a) Step Response Parameters

b) Frequency Response Parameters

FIGURE 14.16. First- and second-order parameters.

Equation 47 – Because the step response is monotonic and asymptotically approaches its final value, there is an infinite wait to see the peak.

Equation 48 – An exponential settles to within about 2% of final value in four time constants.

Equation 49 – The steady-state delay in response to a ramp input is equal to the pole time constant.

Equation 50 – The frequency response of a first-order system rolls off monotonically from its DC value. Hence, the peak of the frequency response occurs at zero frequency.

14.11.2 FORMULAS FOR SECOND-ORDER LOW-PASS SYSTEMS

Here, assume a transfer function of the form

$$H(s) = \left[\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1\right]^{-1}. \tag{51}$$

Then the following relationships hold:

$$t_r \approx 2.2\tau = \frac{2.2}{\omega_h}, \tag{52}$$

$$P_o = 1 + \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right), \tag{53}$$

$$t_p = \frac{T_{osc}}{2} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}, \tag{54}$$

$$t_s\big|_{2\%} \approx 4\tau_{env} = \frac{4}{\zeta\omega_n}, \tag{55}$$

$$\varepsilon_1 = \frac{2\zeta}{\omega_n}, \tag{56}$$

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}}, \quad \zeta < \frac{1}{\sqrt{2}}, \tag{57}$$

$$\omega_p = \omega_n\sqrt{1-2\zeta^2}, \quad \zeta < \frac{1}{\sqrt{2}}, \tag{58}$$

$$\omega_h = \omega_n\left[1 - 2\zeta^2 + \sqrt{2 - 4\zeta^2 + 4\zeta^4}\right]^{1/2} = \omega_n\big|_{\zeta=1/\sqrt{2}}. \tag{59}$$

Commentary and Explanations

Equation 52 – The risetime of a second-order low-pass system is somewhat dependent on the damping ratio. In the limit of zero damping, the product of bandwidth and risetime can be as small as about 1.6. However, for any reasonably well-damped system, the product will be closer to 2.2.

Equation 53 – The peak of the step response overshoot cannot exceed 100% for a second-order low-pass system.

Equation 54 – The time at which the step response peak overshoot occurs is simply one half the ringing period. Recall that the ringing frequency is equal to the imaginary part of the complex pole pair. The formula for t_p follows directly from these two facts.

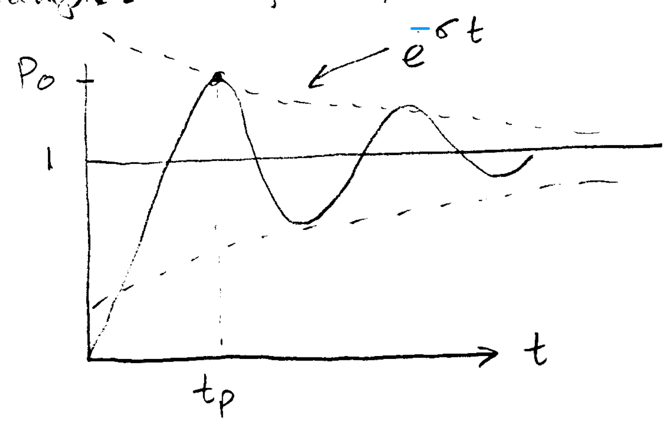
Equation 55 – Just as the imaginary part of the pole frequency controls the oscillatory part of the response, the real part controls the decay. As in the first-order

Why is the 2 pole case of much interest?

Stable feedback systems must behave as one or two pole systems near the crossover frequency.

Many of the relationships that apply to 2nd order systems can be applied to a broader class of systems.

Example: step response overshoot:



$$t_p = \frac{T_{osc}}{2} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$P_o = 1 + \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

$$\sigma = \zeta\omega_n$$

Transient Response : step

$$H(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n}s + 1}$$

simple second-order lowpass

$Y(s) = H(s)U(s)$ for step, $U(s) = \frac{1}{s}$

Rewrite $H(s)$ using $\sigma = \zeta\omega_n$ and $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$H(s) = \frac{\omega_n^2}{(s+\sigma)^2 + \omega_d^2}$$

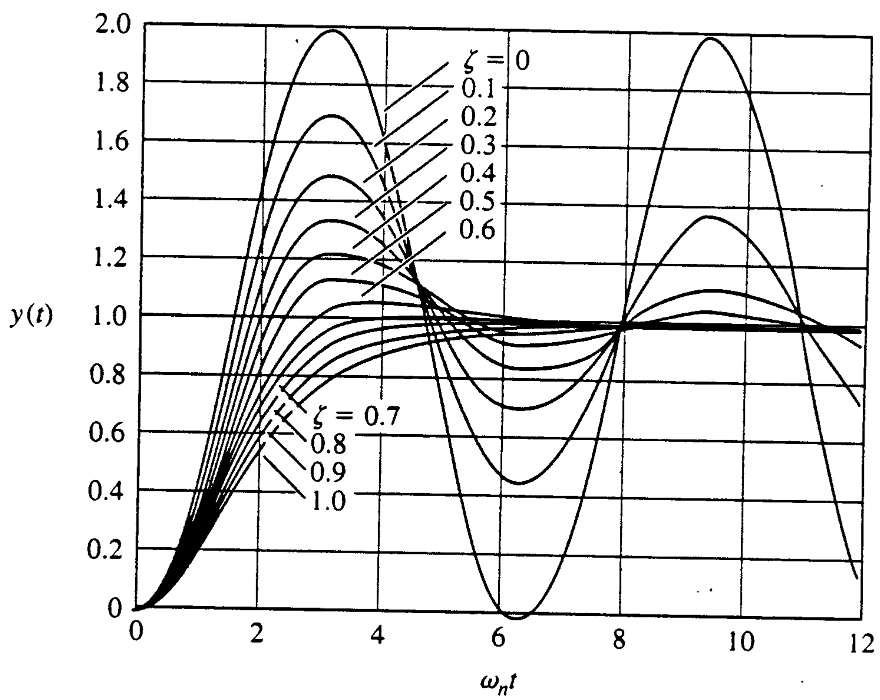
Inverse LT of $\frac{H(s)}{s}$:

$$y(t) = 1 - e^{-\sigma t}\left(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t\right)$$

Look at plot of step response :

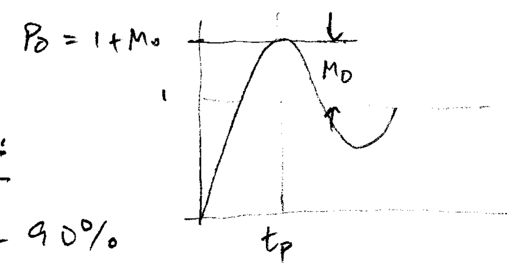
Step Response of $H(s) = \dfrac{1}{\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_n}\right) + 1}$

$$y(t) = 1 - e^{-\sigma t}\left(\cos \omega_d t + \frac{\sigma}{\omega_d} \sin \omega_d t\right)$$



$\sigma = \omega_n \zeta \qquad \omega_d = \omega_n \sqrt{1 - \zeta^2}$

From: G. Franklin, J. Powell, A. Emami-Naeini, Feedback Control of Dynamic Systems, 3rd Ed., Addison-Wesley, 1994.



Time domain specifications:

risetime $t_r = t_{10-90\%}$

overshoot $M_0 = P_0 - 1$

settling time t_s

1. risetime. From the curves, we could take an average, say for $\zeta = 0.5$.

$$t_r \cong \frac{1.8}{\omega_n} \quad \text{or} \quad \frac{2.2}{\omega_n}$$

2. overshoot. take derivative of $y(t)$. set equal to zero.

peak occurs when $\sin \omega_d t = 0$

$$\frac{dy(t)}{dt} = e^{-\sigma t}\left[\frac{\sigma^2}{\omega_d}\sin(\omega_d t) + \omega_d \sin(\omega_d t)\right] = 0$$

$$\omega_d t_p = \pi$$

$$y(t_p) = P_0 = 1 + M_0 = 1 + e^{-\sigma\pi/\omega_d}$$

$$M_0 = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \qquad 0 \le \zeta \le 1$$

3. <u>Settling time.</u> Determined by envelope $e^{-\sigma t}$.

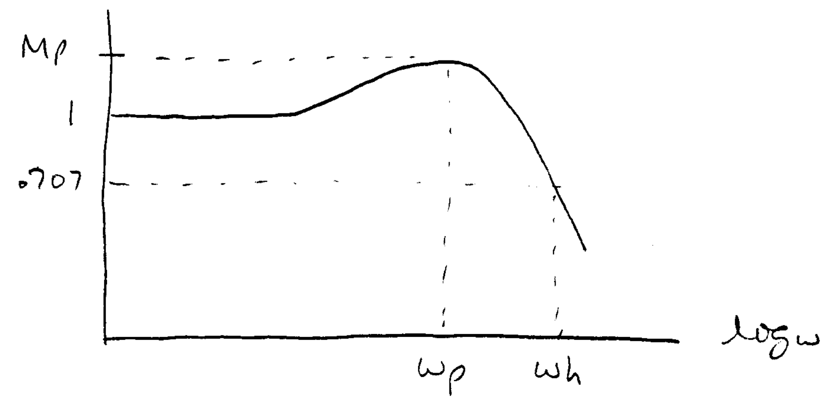
1% settling time.

$$e^{-\zeta\omega_n t_s} = 0.01$$

$$t_s = \frac{4.6}{\zeta\omega_n}$$

These relationships are not so hard to remember and provide guidance for design of the FB dynamic response.

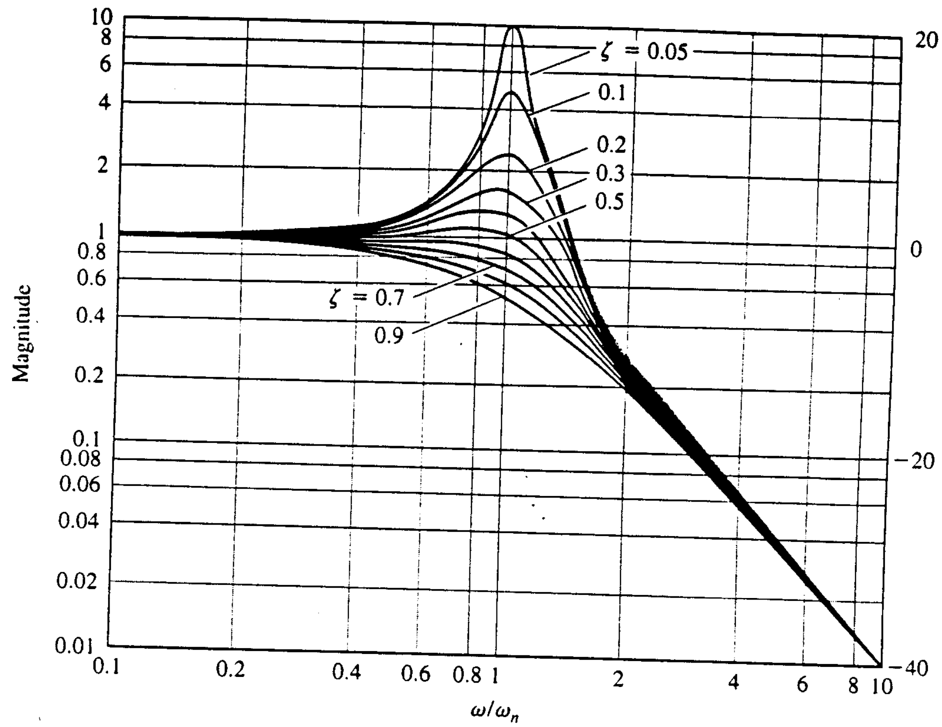
Gain peaking:



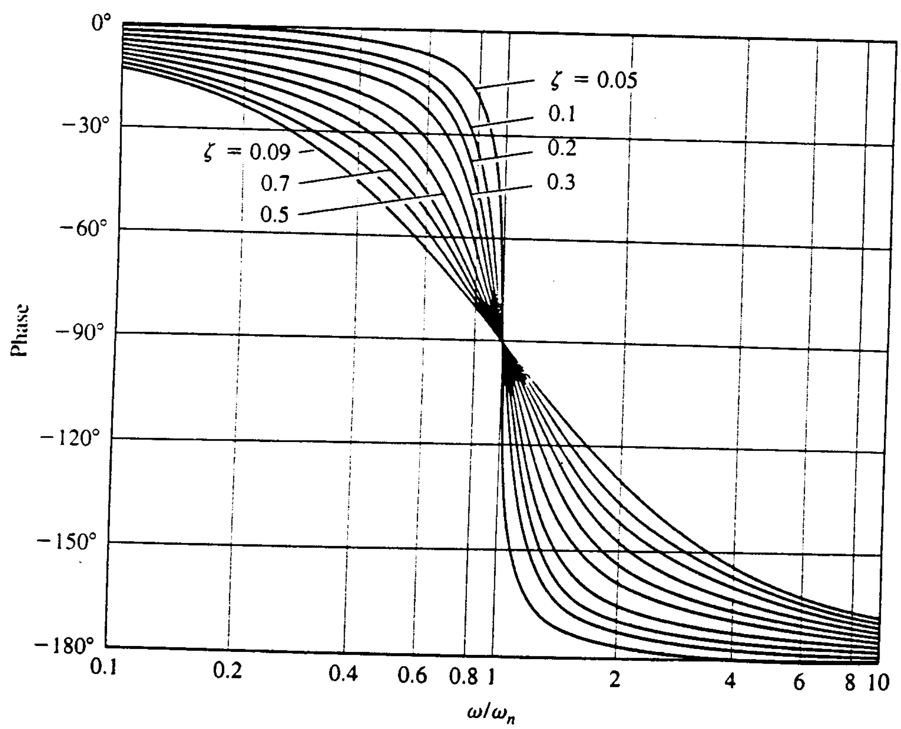
$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \qquad \zeta < \frac{1}{\sqrt{2}}$$

$$\omega_p = \omega_n \sqrt{1-2\zeta^2} \qquad \zeta < \frac{1}{\sqrt{2}}$$

$$\omega_h = \omega_n \left[1 - 2\zeta^2 + \sqrt{2 - 4\zeta^2 + 4\zeta^4}\right]^{1/2}$$



(a)



(b)

Ref. Franklin, op. cit.

Two pole frequency and step response. Low pass. No zeros.

Frequency Response	
3 dB bandwidth	$\omega_{3dB} = \omega_n \sqrt{1 - 2\zeta^2 + \sqrt{2 - 4\zeta^2 + 4\zeta^4}}$
Gain peaking	$M_P = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}} \qquad \zeta < 0.707$
Step Response	
Risetime (10-90%)	$t_r = 2.2 / \omega_{3dB}$
Overshoot (%)	$100 \exp\left(\dfrac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)$
Ringing frequency	$\omega_d = \omega_n \sqrt{1-\zeta^2}$
Settling time	$t_S = -\dfrac{1}{\zeta\omega_n} \ln\left(\dfrac{\%}{100}\right)$

Zero in Loop Gain, T(s).

There will be some cases where we will want to add a zero to the loop gain T(s). How does this zero affect the transient response?

Let's locate the zero frequency relative to the real part of the closed loop pole location using α as a proportionality factor

$$s = -\alpha\zeta\omega_n .$$

$$H(s) = \frac{(s/\alpha\zeta) + 1}{(s/\omega_n)^2 + 2\zeta s/\omega_n + 1}$$

A large α will place the zero far to the left of the poles. Let's normalize $\omega n = 1$. Then,

$$H(s) = \frac{s/\alpha\zeta + 1}{s^2 + 2\zeta s + 1}$$

Split this into 2 equations.

$$H(s) = \frac{1}{s^2 + 2\zeta s + 1} + \frac{1}{\alpha\zeta}\ \frac{s}{s^2 + 2\zeta s + 1}$$

We see that the second term is the derivative of the first term (first term is multiplied by s times a coefficient). This can produce a bump in the step response. See the next figure from G. Franklin, et al, "Feedback Control of Dynamic Systems," 3rd edition, Addison-Wesley, 1994.

Ho(s) is the first term; Hd(s) is the derivative term. We see that if α is close to 1, we get a big increase in the overshoot.

LHP
zero

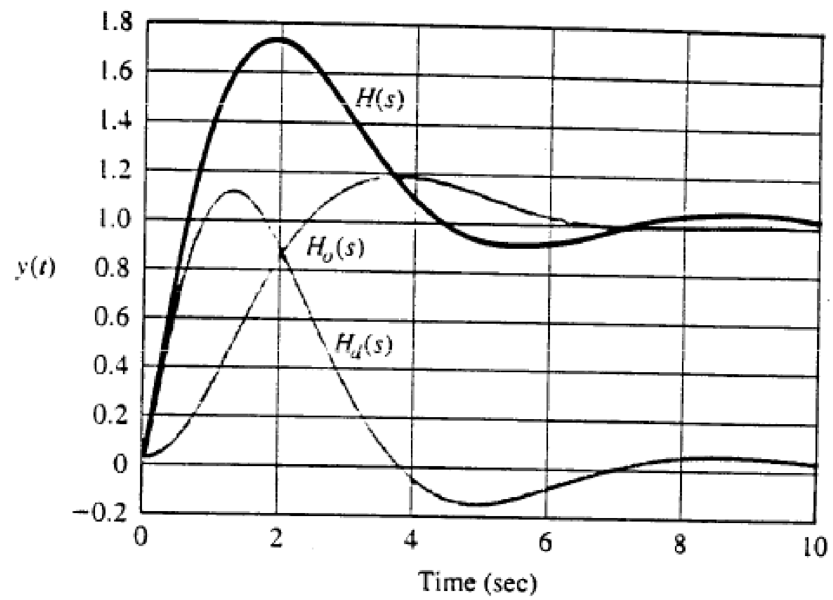
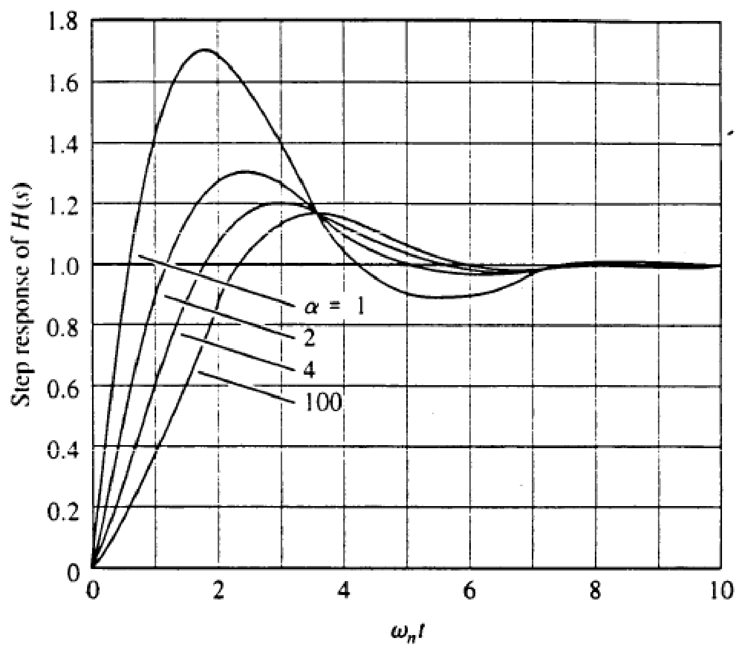
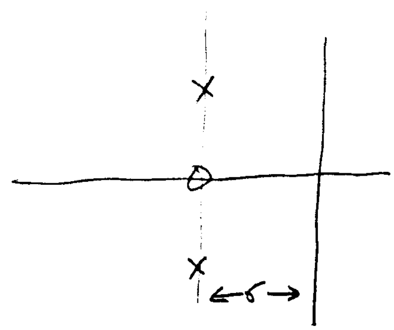


FIGURE 3.20
Plots of the step response of a second-order system with a zero ($\zeta = 0.5$)

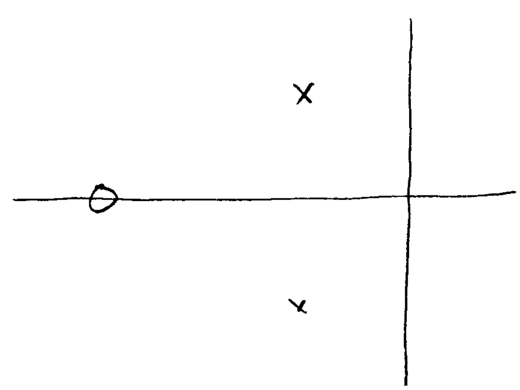


Ref. Franklin, op.cit.

We see that $\alpha = 1$ causes a bump in the step response. This is the case where the zero is at the same location as the real part of the poles.

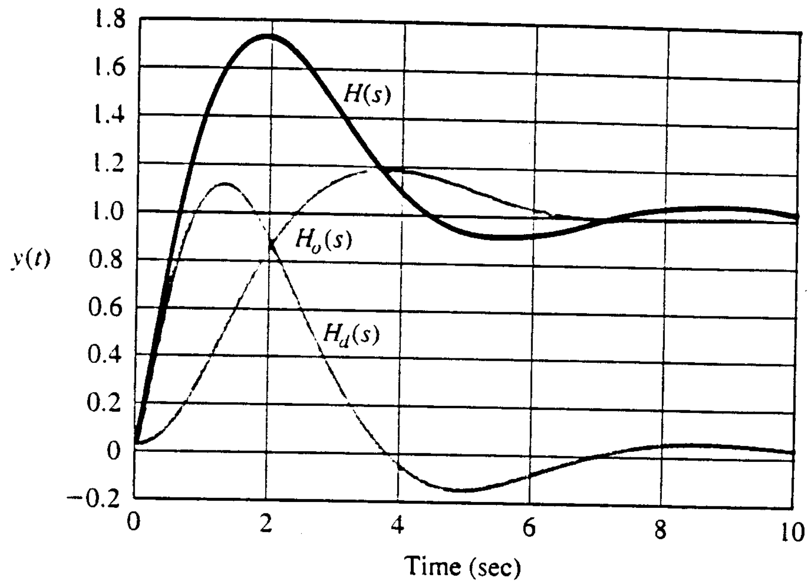


We see that this increases the overshoot. The bump is caused by the derivative of $H_0(s)$. For good transient response, make sure $\alpha > 2$.

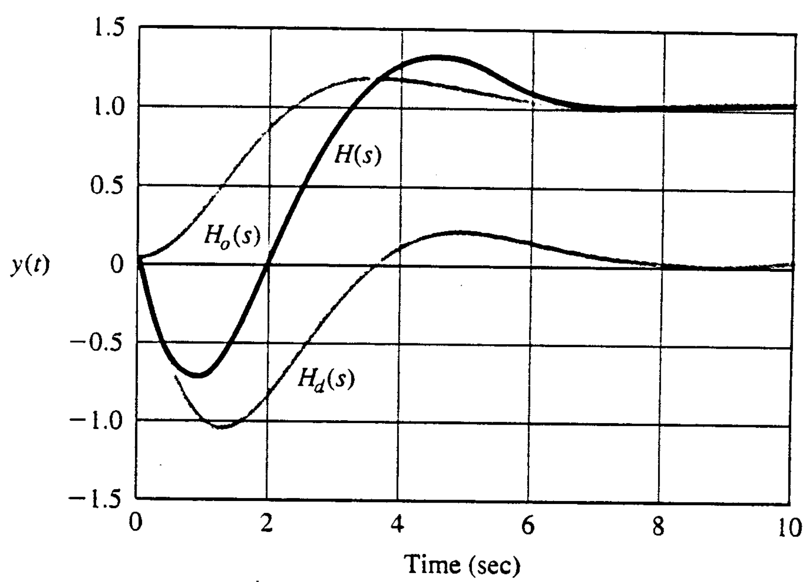


Step response

LHP
zero



RHP
zero



Franklin, op. cit.

Compensation

Question: What happens when the phase margin is too small or negative for the particular value of feedback required for an application.

- The transient response will ring,
- gain will peak,
- or possibly oscillation.

If you are building an oscillator, that might be good, but if you intended it as an amplifier then you must modify its frequency response to make it useable. Compensation is a technique that accomplished this, albeit at the expense of bandwidth.

Many techniques are available:

- Add a dominant pole
- Move a dominant pole
- Miller compensation
- Add a zero to the closed loop gain

1. Dominant Pole Compensation

Add another pole that is much lower in frequency than the existing poles of the amplifier or system. This is the least efficient of the compensation techniques but may be the easiest to implement.

- Reduces bandwidth,
- but increases the phase margin at the crossover frequency.
- |T| is reduced over the useful bandwidth,

Thus, some of the feedback benefits are sacrificed in order to obtain better stability

Extrapolate back from the crossover frequency at 20 dB/decade The frequency, P_D, where the line intersects the open loop gain Ao is the new dominant pole frequency. In this case,

$$|T(j\omega)| = 1 \text{ at } \omega = |p_1|$$

$$45^o \text{ phase margin}$$

Note that we are assuming that the other poles are not affected by the new dominant pole. This is not always the case, and computer simulation will be needed to optimize the design. Nevertheless, this simple method will usually help you get started with a solution that will generally work even though not optimum.

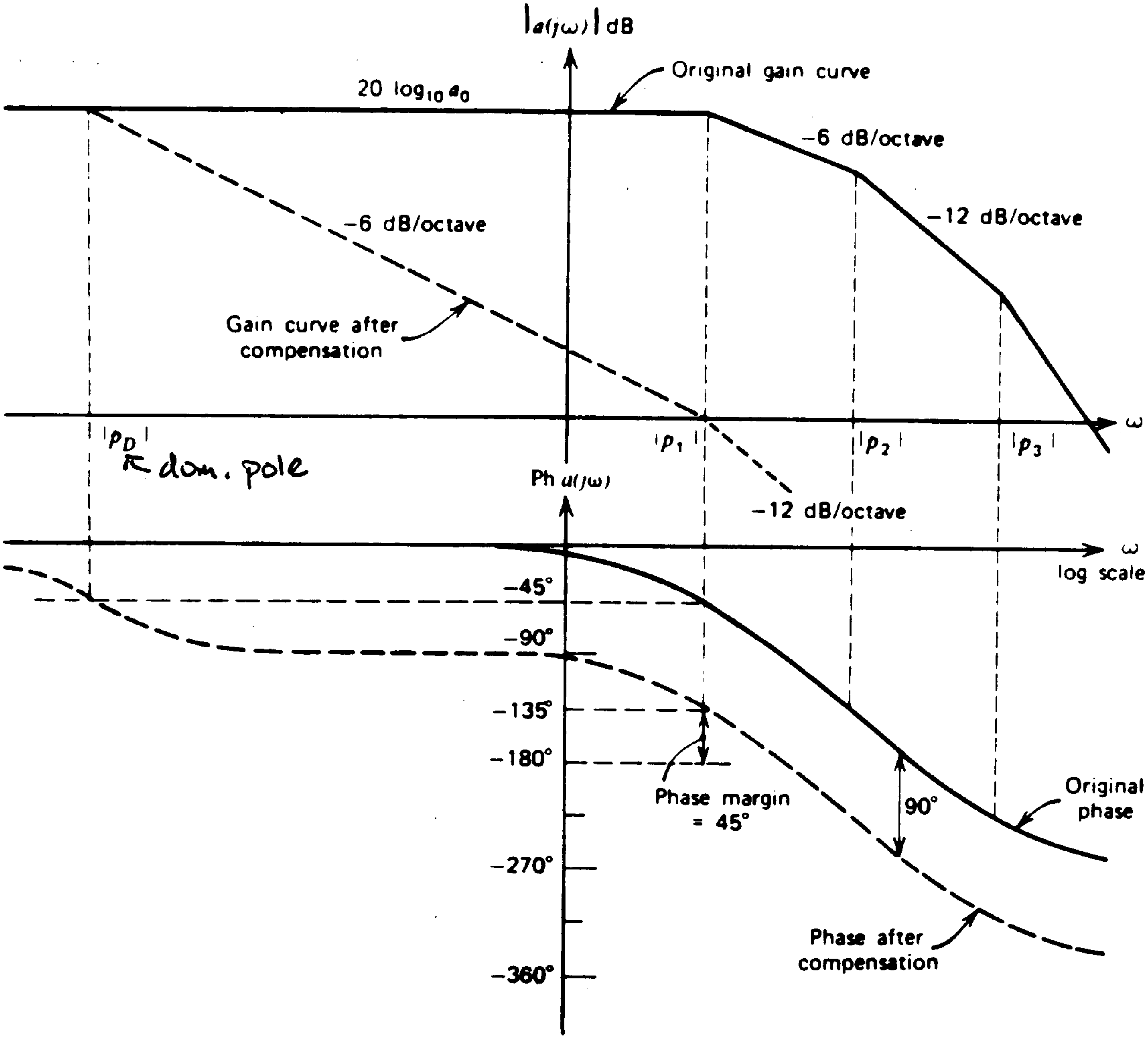
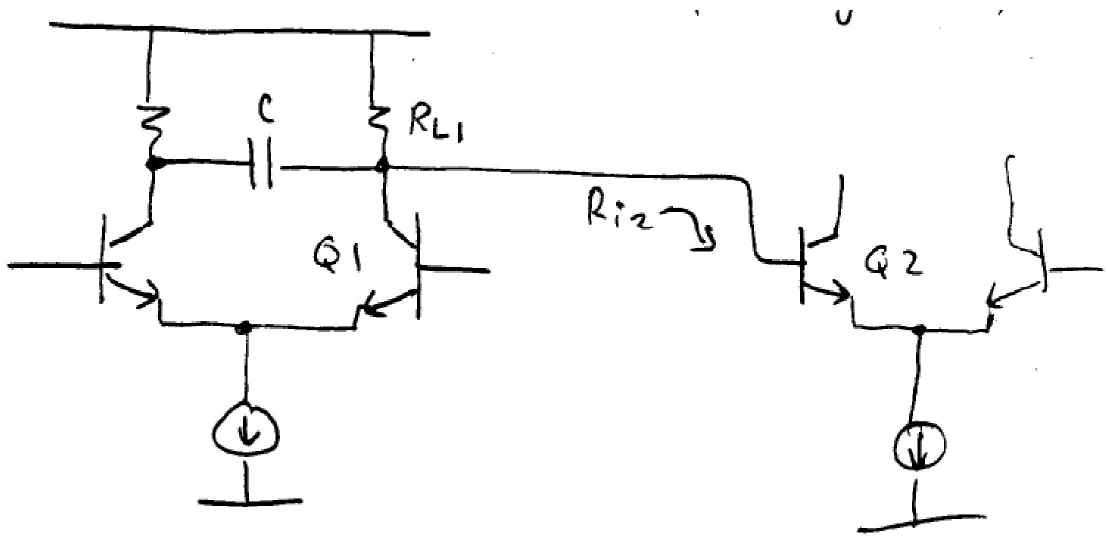


Figure 9.12. Gain and phase versus frequency for a three-pole basic amplifier. Compensation for unity-gain feedback operation ($f = 1$) is achieved by introduction of a negative real pole with magnitude $|p_D|$.

For example, you could add C to produce a dominant pole at

$$p_D = \frac{1}{2C(R_{L1} \| R_{i2})}$$



This approach may require a large compensating capacitor, C.

Let’s look at some better approaches.

2. Reduce |p1| instead of adding yet another pole.

- Retains more bandwidth
- Requires less C to shift an existing pole
- p2 and p3 may even be moved up to a higher frequency

Move p_1 to p_1'such that the new crossover frequency is at $|p_2| >> |p_1|$

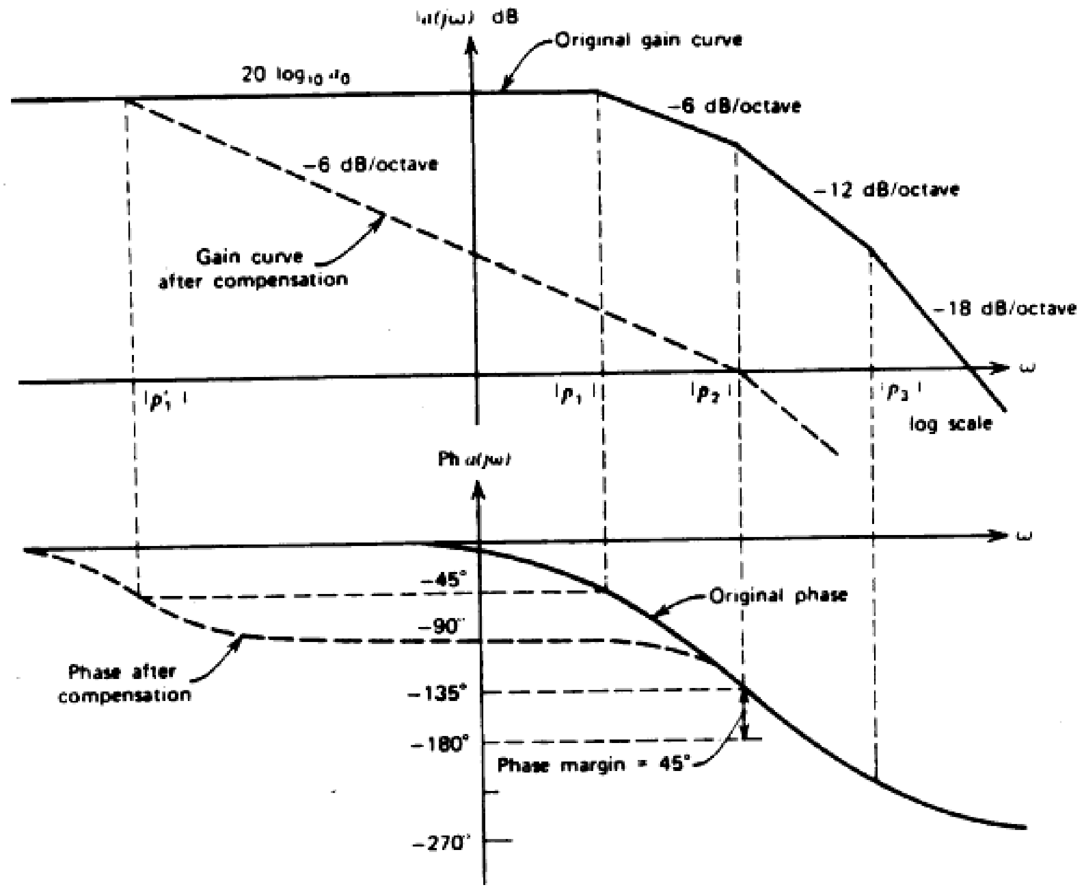


Figure 9.15 Gain and phase versus frequency for an amplifier compensated for use in a feedback loop with $f = 1$ and a phase margin of 45°. Compensation is achieved by reducing the magnitude $|p_1|$ of the dominant pole of the original amplifier.

From Gray, Hurst, Lewis and Meyer, Analysis and Design of Analog Integrated Circuits, 4th ed., J. Wiley, 2001.

3. What if we reduce p1 and increase p2 at the same time!

- Big bandwidth improvement
- Phase margin improves without sacrifice of bandwidth
- Pushes out crossover frequency

MILLER COMPENSATION

The first and second pole frequencies can be estimated by the method of time constants. Assuming that

$$H(s) = \frac{K}{a_2 s^2 + a_1 s + a_0}$$

then,

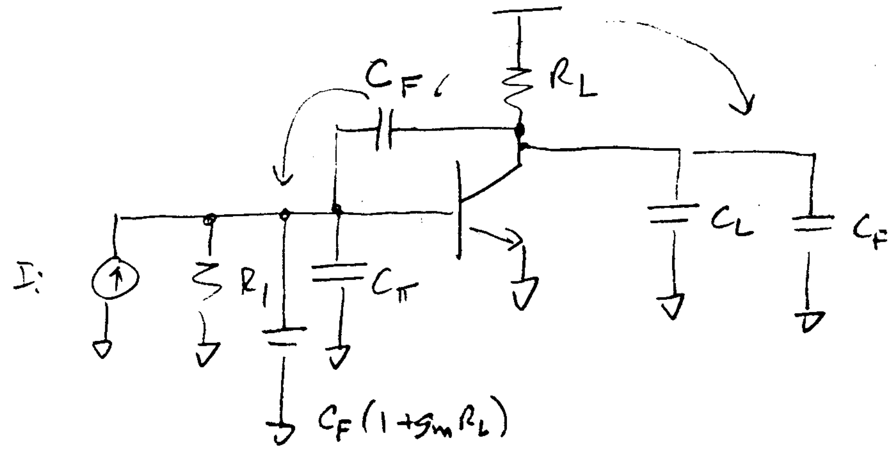
$$p_1 \cong \frac{-1}{a_1}$$

and

$$p_2 \cong \frac{-a_1}{a_2}$$

Miller Compensation

Estimate pole frequencies vs. C_F.



$$a_1 \cong -\frac{1}{p_1} = R_1\left[C_\pi + C_F(1+g_m R_L)\right] + R_L(C_L + C_F)$$

$$\cong R_1 C_F g_m R_L + C_F R_L \qquad \text{for } C_F \gg C_L$$

$$\cong R_1 C_F g_m R_L$$

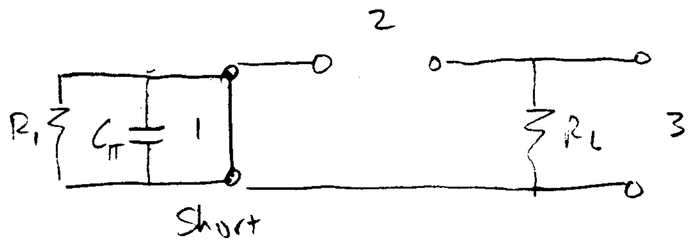
$$p_1 \cong \frac{-1}{g_m R_L C_F R_1}$$ decreases as C_F increases.

Now, estimate a_2:

$$a_2 = R_{11}^{0} C_{\pi} R_{22}^{1} C_F + R_{11}^{0} C_{\pi} R_{33}^{1} C_L + R_{22}^{3} C_F R_{33}^{0} C_L$$

$R_{11}^{0} = R_1$

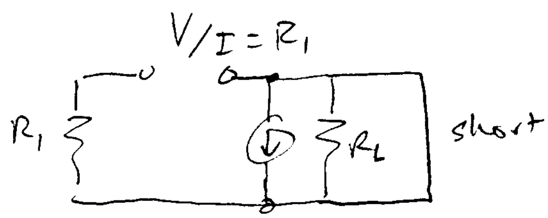
$R_{22}^{1} = R_L$



$R_{33}^{1} = R_L$

$R_{33}^{0} = R_L$

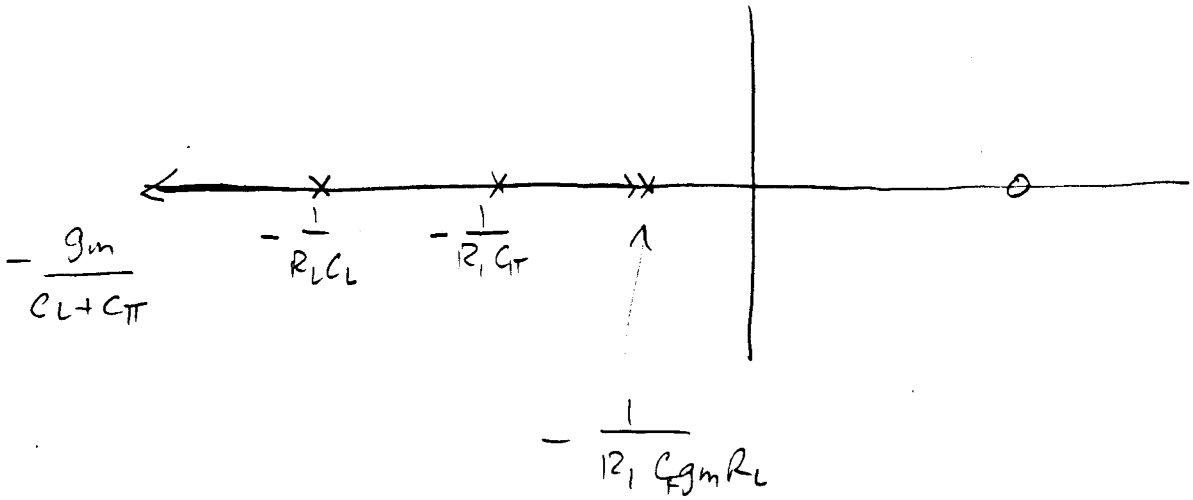
$R_{22}^{3} = R_1$



$$a_2 = R_1 R_L \left[C_F (C_L + C_\pi) + C_\pi C_L \right]$$

$$p_2 \cong -\frac{a_1}{a_2} = \frac{g_m C_F}{C_F (C_L + C_\pi) + C_\pi C_L}$$

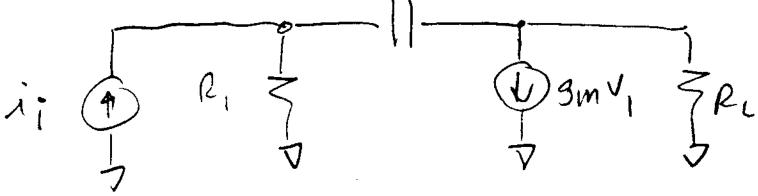
For large C_F $\quad p_2 \rightarrow \dfrac{-g_m}{C_L + C_\pi}$



Now: One caution to consider.

There is a RHP zero in the transfer function

$\xrightarrow{C_F}$ feedforward path. 90° phase delay.



at high frequencies, feedforward current adds extra phase lag, $-90°$.

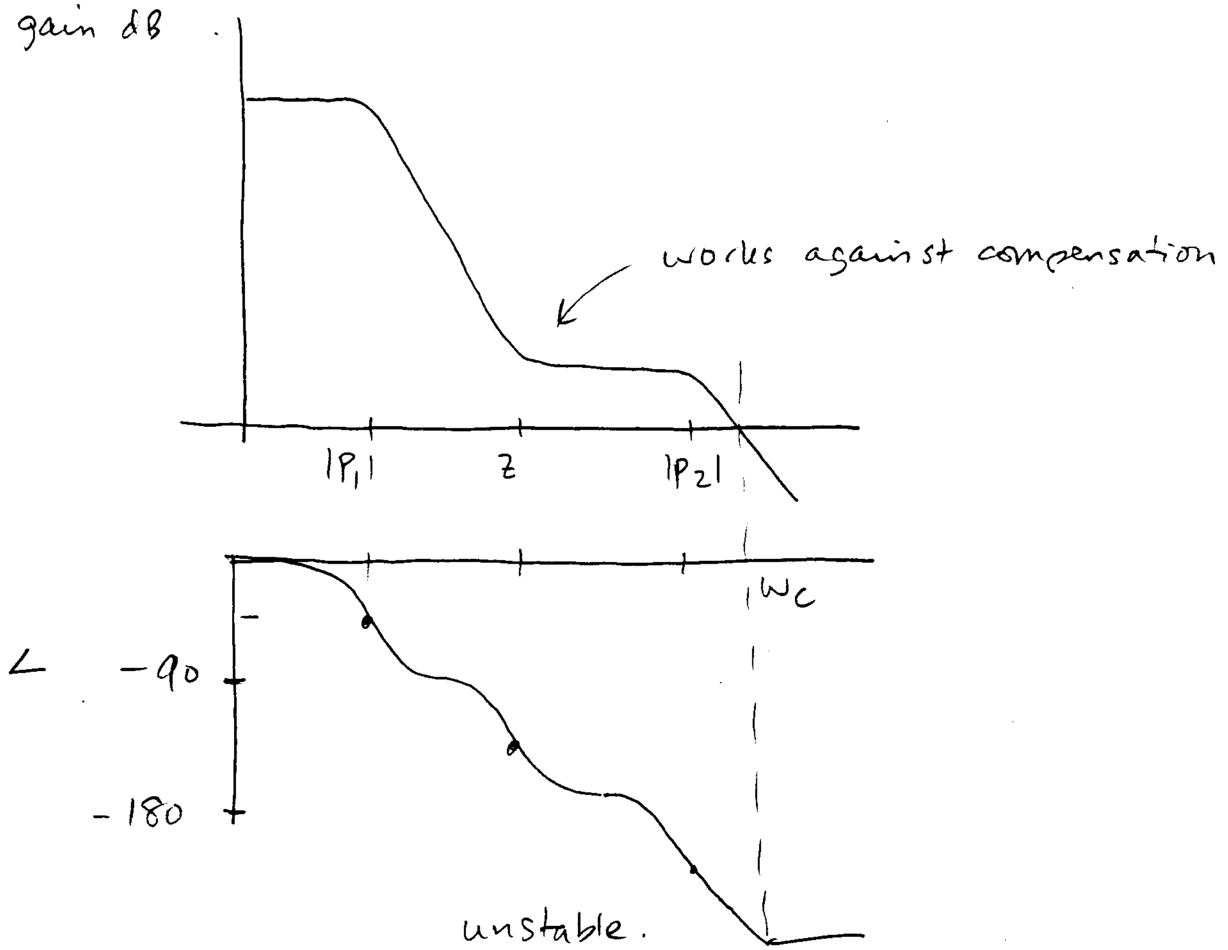
$$\frac{v_o}{i_i} = \frac{(sC_F - g_m) R_1 R_2}{D(s)}$$

$$z = + \frac{g_m}{C_F}$$

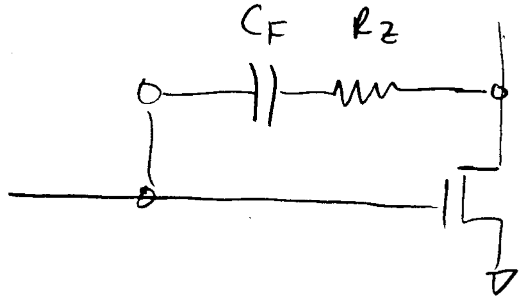
as C_F increases, z decreases and can become a nuisance.

For BJT amp, g_m is generally large.
z can usually be ignored.

For MOSFET amp, $g_{m,MOS} << g_{m,BJT}$



But, FB capacitor can be modified to cancel the zero.



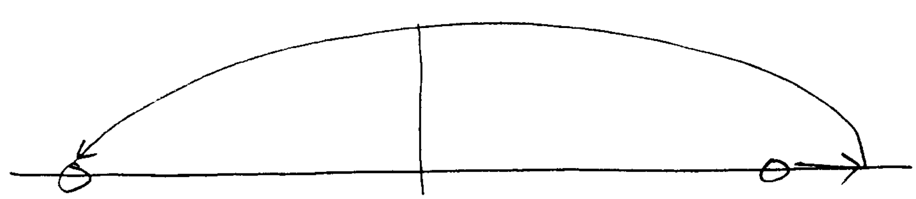
now

$$z = \frac{1}{C_F\left(\frac{1}{g_m} - R_Z\right)}$$

so, if $R_Z = 1/g_m$, it goes away.

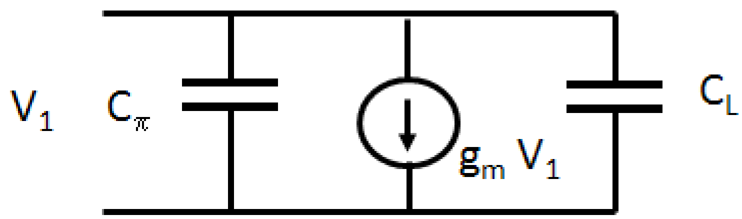
or, if $R_Z > 1/g_m$, moves to LHP.

then it can improve phase margin.



Alternatively, if we assume that the non-dominant pole is at a very high frequency

CF behaves like a short



The current source just looks like:

$$Z = \frac{V_1}{i_1} = \frac{1}{g_m}$$

so,

$$p_2 \cong \frac{-g_m}{C_\pi + C_L}$$

A much easier way to see how the pole splitting comes about....

Compensation by adding a zero in the feedback path

In wideband amplifier applications, adding dominant poles or Miller compensation is undesirable due to the loss in bandwidth that is incurred. In some cases, it is possible to add a zero to the feedback path by adding a frequency dependent component. The zero bends the root locus and improves the bandwidth as well as the damping of the amplifier.

Adding a zero to f(s) will put a zero in T(s) and thus affects the root locus. But, it doesn't add a zero to A(s). Thus, it doesn't produce the overshoot problems that a zero in the forward path would cause. The forward path zero would show up in the closed loop transfer function.

If we consider the overall gain with feedback as:

$$A(s) = \frac{a(s)}{1 + a(s)f(s)}$$

and a(s) = Na(s)/Da(s) and f(s) = Nf(s)/Df(s)

Then,

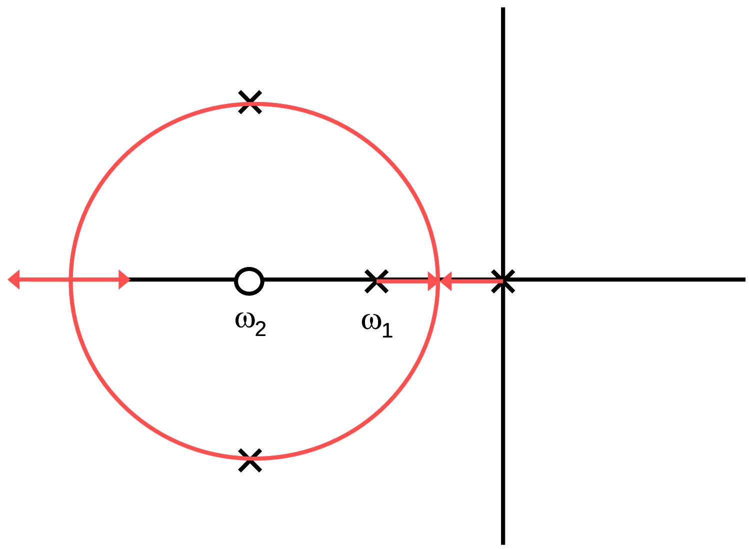
$$A(s) = \frac{a_o N_a(s) D_f(s)}{D_f(s) D_a(s) + T_o N_a(s) N_f(s)}$$

Here we see that the zero Nf(s) shows up in the denominator of A(s) multiplied by To. And, typically for this type of compensation, the pole contributed by the f(s) block is at a much higher frequency than the zero.

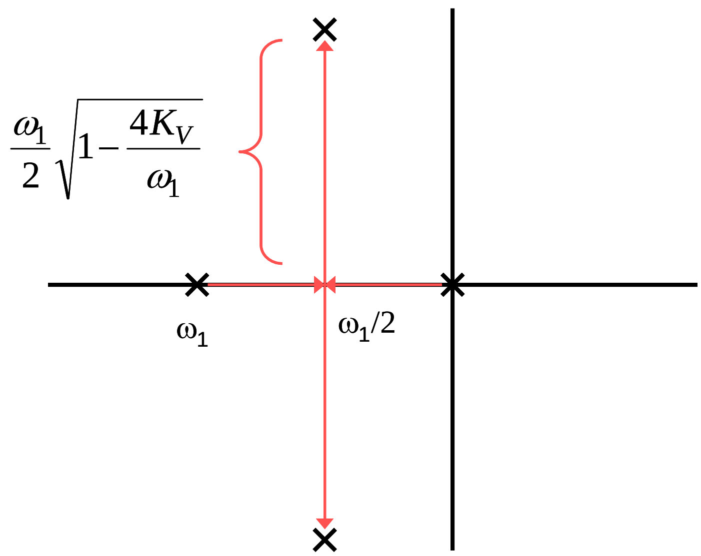
Referring to the circular root locus sketched on the next page, we see that the zero bends the poles away from the jω axis. This increases ζ, improving damping, and also improves bandwidth.

The root locus will travel around the zero in a circle if there are no other poles to the left.

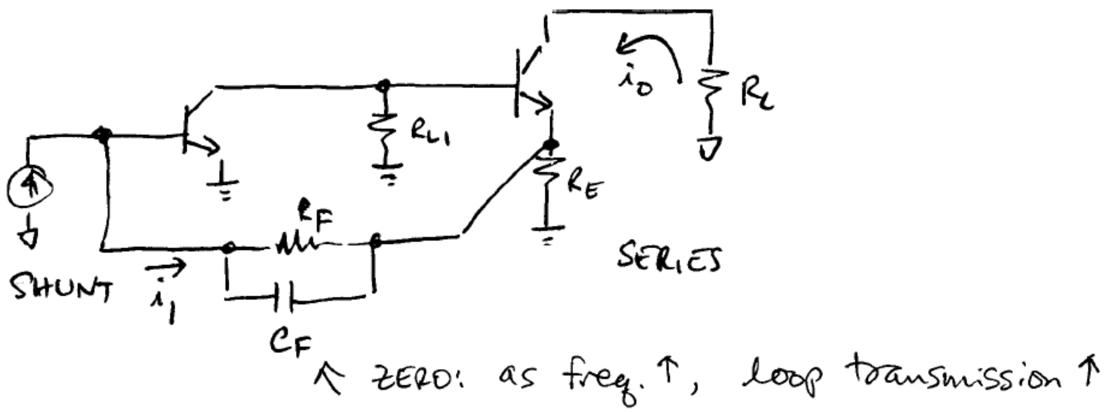
The poles will terminate at large |T| on zeros at ω_2 and at infinity.



We can appreciate the effectiveness of this by comparing with the two pole low pass case where the poles remain at the same distance from the jω axis as |T| is increased.

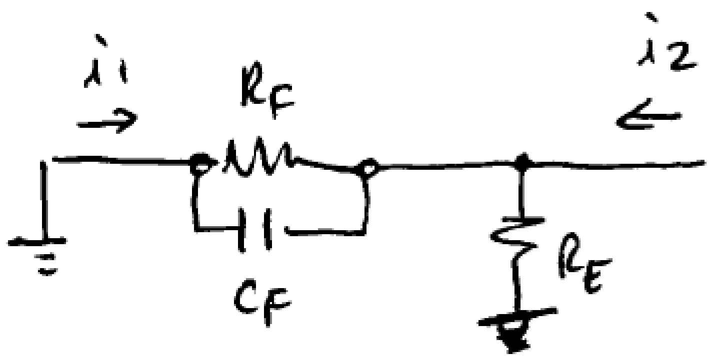


So, how do you implement a zero in the feedback path?



We need to find f(s).

- With SHUNT at input, currents are summed
- With SERIES output, current is sampled

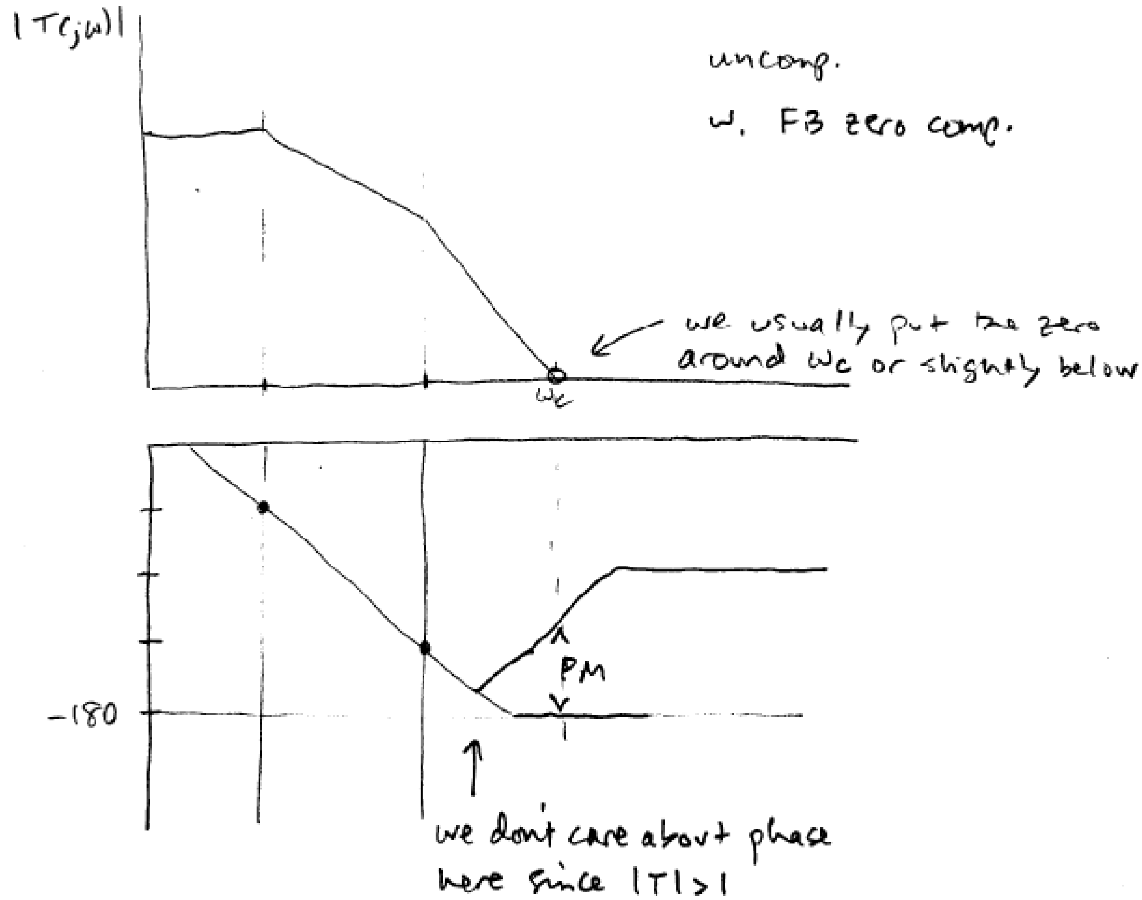


$$f(s) = \left(\frac{i_1}{i_2}\right) = -\left(\frac{R_E}{R_E + R_F}\right)\left(\frac{1 + sR_F C_F}{1 + sC_F \dfrac{R_E R_F}{R_E + R_F}}\right)$$

Zero: $\dfrac{-1}{R_F C_F}$ Pole: $\dfrac{-1}{\left(\dfrac{R_E}{R_E + R_F}\right) C_F R_F}$

Typically, the pole frequency is much higher than the zero, so it does not bend the root locus very much.

Next, let's look at the phase margin.



Recall that

$$A(j\omega)=\frac{a_0}{1+T(j\omega)}$$

This blows up when

$$T(j\omega)=1e^{-j\pi}=-1$$

Thus, even if the phase reaches -180 degrees, if the magnitude is not equal to 1, we do not have instability.