

# Interconnections and Signal Integrity

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*SemChip*

## DAC Tutorial

22, June 2001

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## Future System Needs and Functions

*Auto*



**MEMS**

*Consumer*

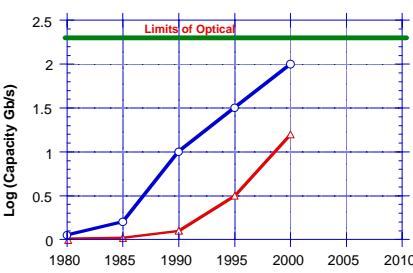
*Digital Wireless*



*Analog, RF  
Computer*



**High-speed Digital**

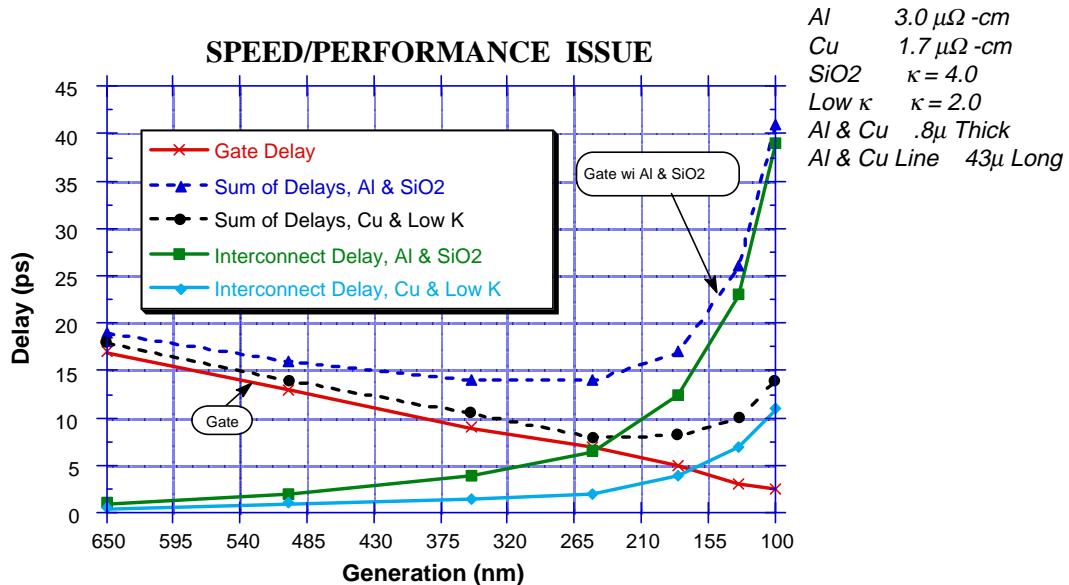


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# The Interconnect Bottleneck



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## Semiconductor Technology Trends

	1997	2003	2006	2012
<b>Chip size (mm<sup>2</sup>)</b>	300	430	520	750
<b>Number of transistors (million)</b>	11	76	200	1400
<b>Interconnect width (nm)</b>	200	100	70	35
<b>Total interconnect length (km)</b>	2.16	2.84	5.14	24

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# The Interconnect Bottleneck

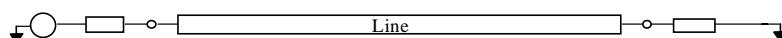
Technology Generation	MOSFET Intrinsic Switching Delay	Response Time
1.0 um	~ 10 ps	~ 1 ps
0.01 um	~ 1 ps	~ 100 ps

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## Chip-Level Interconnect Delay

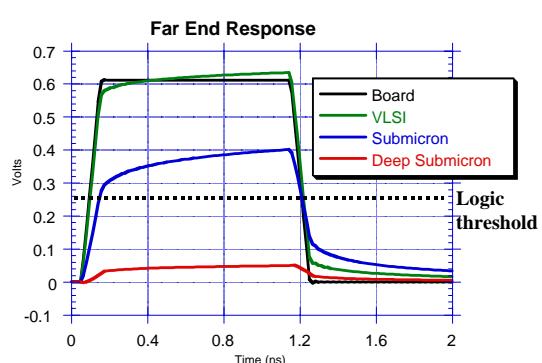
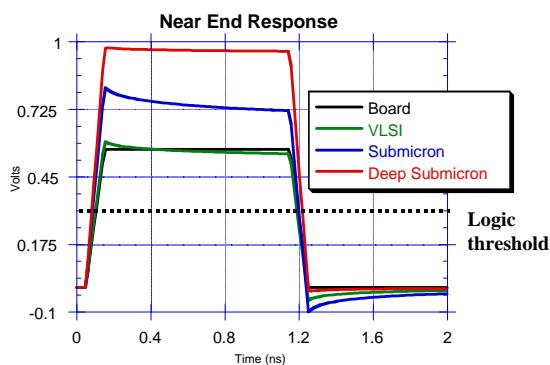


### Pulse Characteristics:

rise time: 100 ps  
fall time: 100 ps  
pulse width: 4ns

### Line Characteristics

length : 3 mm  
near end termination: 50 Ω  
far end termination 65 Ω



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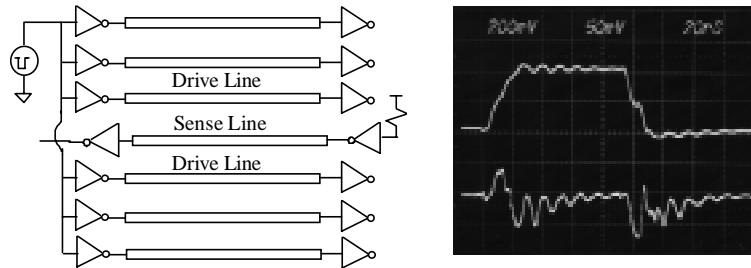
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# Interconnect Bottleneck

## Signal Integrity

Crosstalk	Dispersion	Attenuation
Reflection	Distortion	Loss
Delta I Noise	Ground Bounce	Radiation

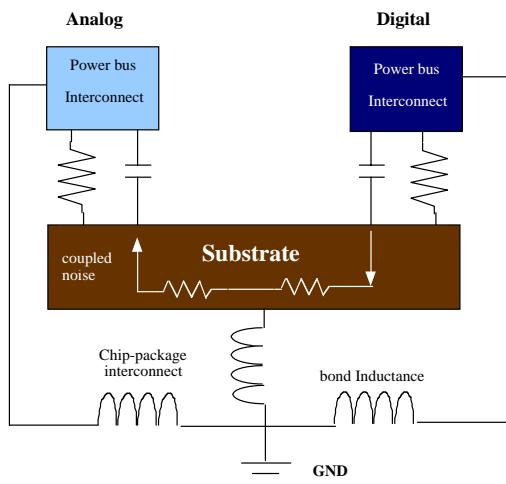


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## Mixed Signal Noise



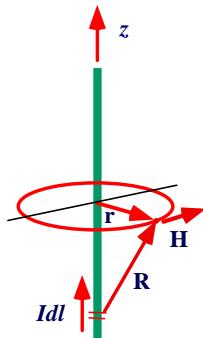
- Simultaneous switching and inductance ( $L_{eff}$ )
- $L_{eff}$  is f( current magnitude and direction)
- Interactions between noise generated by power/ground and signal paths

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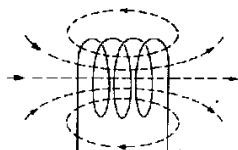
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# INDUCTANCE



$$\text{Inductance} = \frac{\text{Total flux linked}}{\text{Current}}$$

# INDUCTANCE

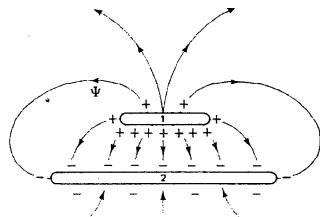


$$L = \frac{N\Phi}{I}$$

$N$  : number of turns

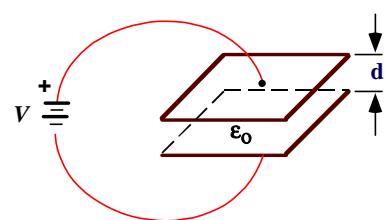
$\Phi$  : flux per turn

# CAPACITANCE



$$\text{Capacitance} = \frac{\text{Total charge}}{\text{Voltage}}$$

# CAPACITANCE



$$C = \frac{\epsilon_0 A}{d}$$

A : area

$\epsilon_0$  : permittivity

# Package Inductance & Capacitance

<u>Component</u>	<u>Capacitance</u> (pF)	<u>Inductance</u> (nH)
<b>68 pin plastic DIP pin<sup>†</sup></b>	<b>4</b>	<b>35</b>
<b>68 pin ceramic DIP pin<sup>††</sup></b>	<b>7</b>	<b>20</b>
<b>68 pin SMT chip carrier<sup>†</sup></b>	<b>2</b>	<b>7</b>

<sup>†</sup> No ground plane; capacitance is dominated by wire to wire component.

<sup>††</sup> With ground plane; capacitance and inductance are determined by the distance between the lead frame and the ground plane, and the lead length.

# Package Inductance & Capacitance

<u>Component</u>	<u>Capacitance</u> (pF)	<u>Inductance</u> (nH)
<b>68 pin PGA pin<sup>††</sup></b>	<b>4</b>	<b>35</b>
<b>256 pin PGA pin<sup>††</sup></b>	<b>7</b>	<b>20</b>
<b>Wire bond</b>	<b>1</b>	<b>1</b>
<b>Solder bump</b>	<b>0.5</b>	<b>0.1</b>

<sup>†</sup> No ground plane; capacitance is dominated by wire to wire component.

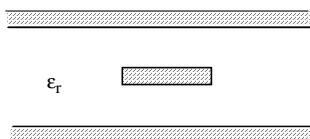
<sup>††</sup> With ground plane; capacitance and inductance are determined by the distance between the lead frame and the ground plane, and the lead length.

# CONDUCTIVITY OF DIELECTRIC MATERIALS

<u>Material</u>	<u>Conductivity (<math>\Omega^{-1} m^{-1}</math>)</u>
Germanium	2.2
Silicon	0.0016
Glass	$10^{-10} - 10^{-14}$
Quartz	$0.5 \times 10^{-17}$

$$\text{Loss TANGENT : } \tan\delta = \frac{\sigma}{\omega \epsilon}$$

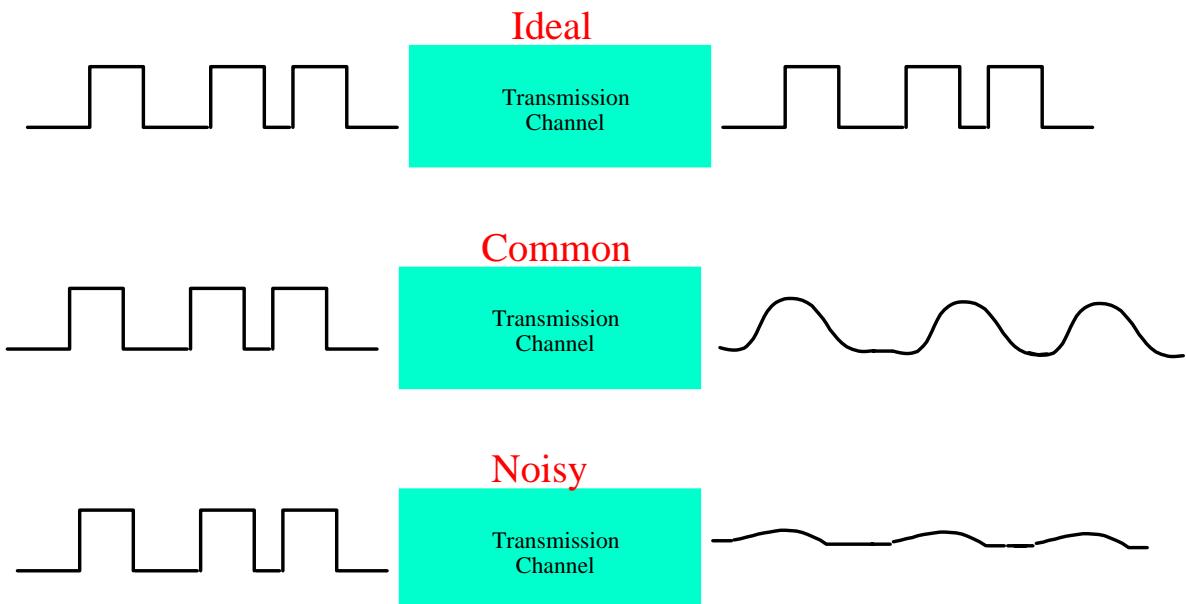
## Propagation Speeds



$$v = \frac{c}{\sqrt{\epsilon_r}}$$

<u>Dielectric</u>	<u><math>\epsilon_r</math></u>	<u>v (cm/ns)</u>
Polymide	2.5-3.5	16-19
Silicon Dioxide	3.9	15
Epoxy Glass (PC board)	5	13
Alumina (ceramic)	9.5	10

# Signal Integrity

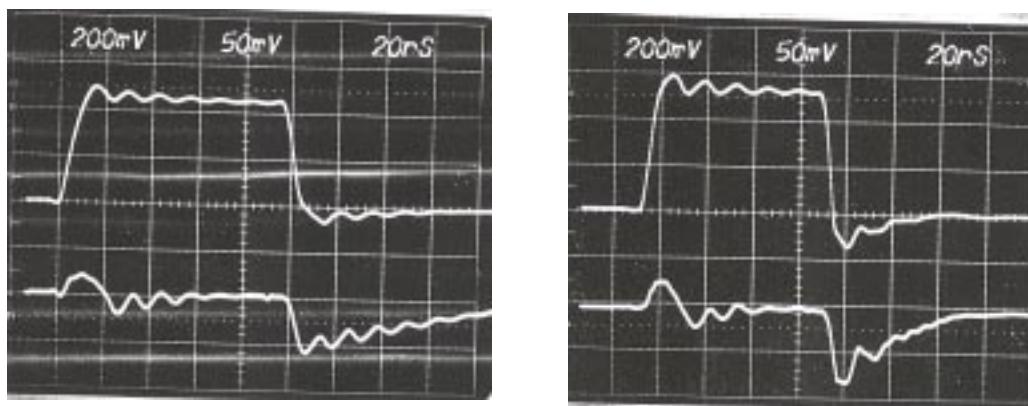


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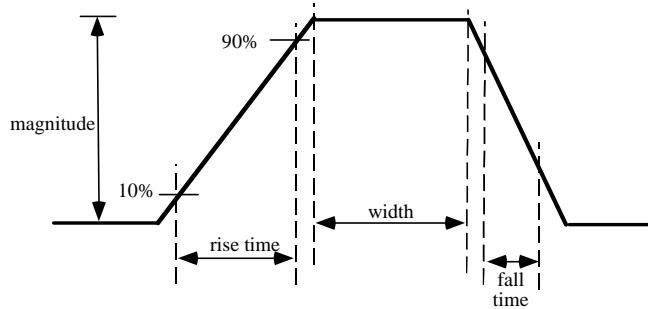
## Signal Degradation



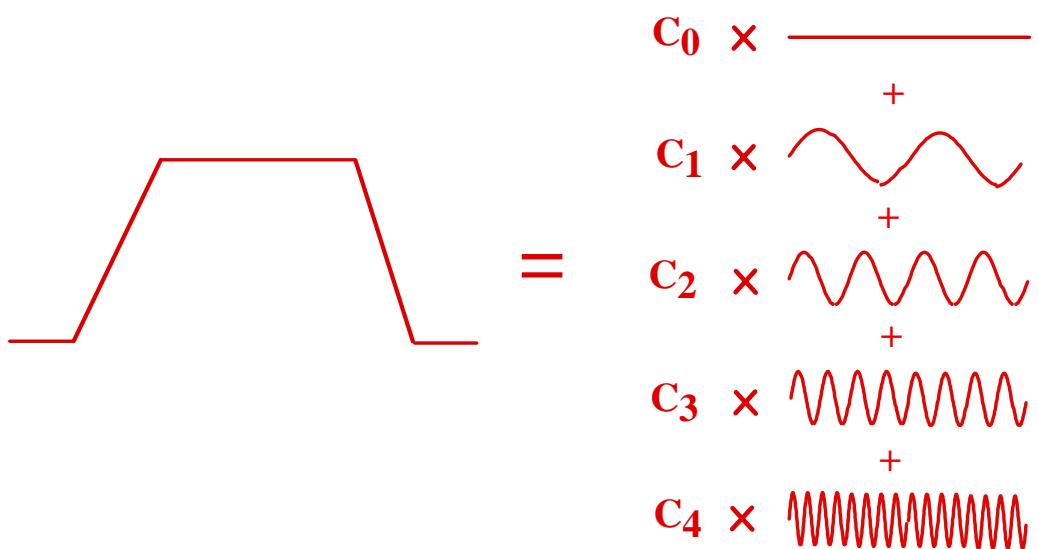
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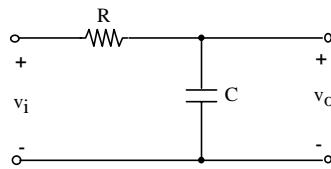
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## Frequency Components of Digital Signal



# RC Network

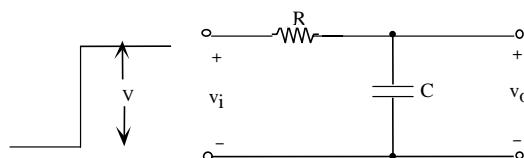


**A** is in the steady state gain of the network;  $A = \frac{v_o(f)}{v_i(f)}$

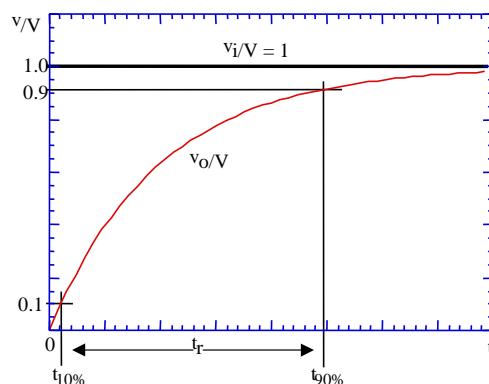
$$|A| = \frac{1}{\sqrt{1+(f/f_2)^2}} \quad f_2 = \frac{1}{2\pi RC}$$

The gain falls to 0.707 of its low-frequency value at the frequency  $f_2$ .  $f_2$  is the *upper 3-dB frequency* or the 3-dB bandwidth of the RC network.

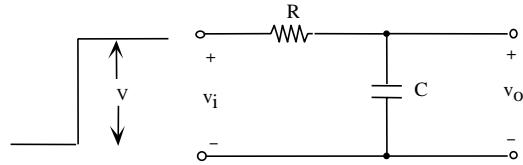
# RC Network



$$v_o = V(1 - e^{-t/RC})$$



# RC Network

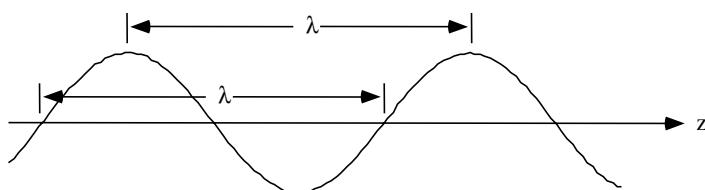


**Rise time :**  $t_r = t_{90\%} - t_{10\%}$

$$t_r = 2.2RC = \frac{2.2}{2\pi f_2} = \frac{0.35}{f_2}$$

**Rule of thumb :** A 1-ns pulse  
requires a circuit with a 3-dB  
bandwidth of the order of 2 GHz.

## WAVE PROPAGATION



**Wavelength :**  $\lambda$

$$\lambda = \frac{\text{propagation velocity}}{\text{frequency}}$$

# Why Transmission Lines ?

In Free Space

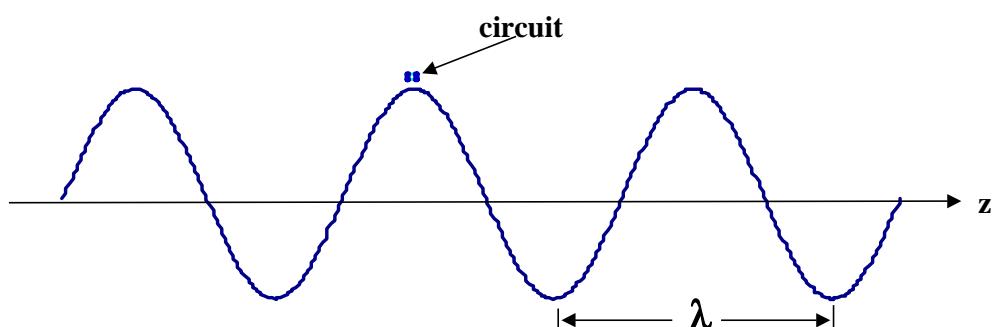
At 10 KHz :  $\lambda = 30 \text{ km}$

At 10 GHz :  $\lambda = 3 \text{ cm}$

Transmission line behavior is prevalent when the structural dimensions of the circuits are comparable to the wavelength.

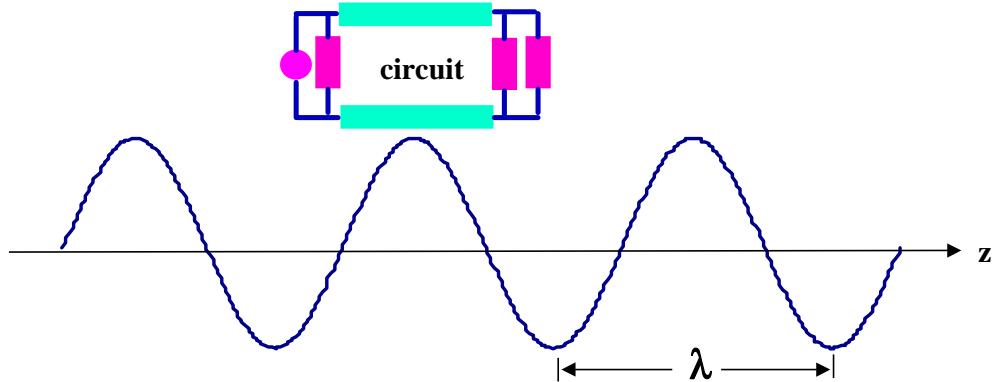
## Transmission Line Model

Let  $d$  be the largest dimension of a circuit



If  $d \ll \lambda$ , a lumped model for the circuit can be used

# Transmission Line Model



If  $d \approx \lambda$ , or  $d > \lambda$  then use transmission line model

## Frequency Dependence of Lumped Circuit Models

At higher frequencies, a lumped circuit model is no longer accurate for interconnects and one must use a distributed model. Transition frequency depends on the dimensions and relative magnitude of the interconnect parameters.

$$f \approx \frac{0.3 \times 10^9}{10d\sqrt{\epsilon_r}} \quad t_r \approx \frac{0.35}{f}$$

# Lumped Circuit or Transmission Line?

A) Determine frequency or bandwidth of the signal

-Microwave:  $f = \text{operating frequency}$

-Digital:  $f = \frac{0.35}{\text{rise time}}$

B) Determine propagation velocity in medium,  $v$ ,

next calculate wavelength  $\lambda = \frac{v}{f}$

# Lumped Circuit or Transmission Line?

C) Compare wavelength with dimensions (feature size)  $d$ .

Case 1: If  $\lambda \gg d$  use lumped circuit equivalent

Total inductance =  $L \times \text{length}$

Total capacitance =  $C \times \text{length}$

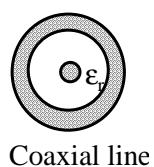
Case 2: If  $\lambda \approx 10d$  or  $\lambda < 10d$ , use transmission-line model

# Frequency Dependence of Lumped Circuit Models

	Dimension	Frequency	Rise time
<b>Printed circuit line (epoxy, glass)</b>	<b>10 in</b>	<b>&gt;55 MHz</b>	<b>&lt;7 ns</b>
<b>Package lead frame (ceramic)</b>	<b>1 in</b>	<b>&gt;400 MHz</b>	<b>&lt;0.9 ns</b>
<b>VLSI interconnection* (silicon)</b>	<b>100 <math>\mu</math>m</b>	<b>&gt;8 GHz</b>	<b>&lt;50 ps</b>

\* Using RC criterion for distributed effect

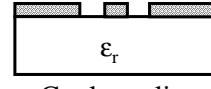
## Types of Transmission Lines



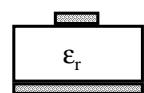
Coaxial line



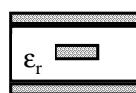
Waveguide



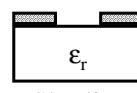
Coplanar line



Microstrip

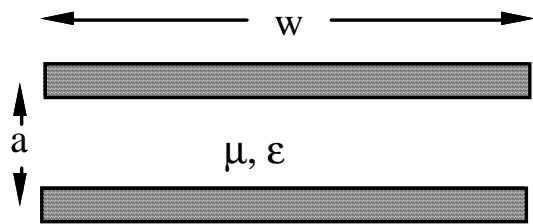


Stripline



Slot line

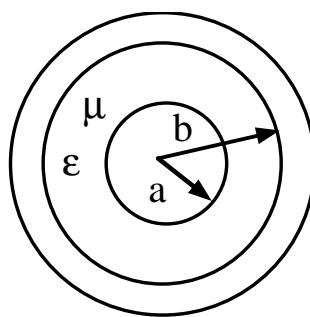
## Parallel-plate Transmission Line



$$L = \frac{\mu a}{w}$$

$$C = \frac{\epsilon w}{a}$$

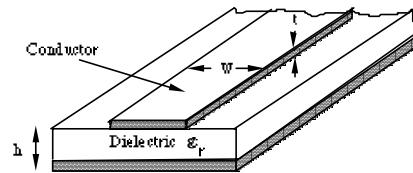
## Coaxial Transmission Line



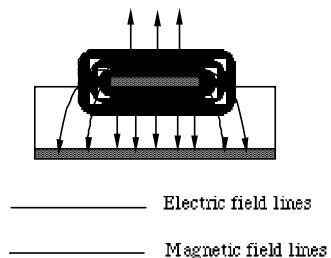
$$L = \mu \ln \frac{b}{a}$$

$$C = \frac{2\pi\epsilon}{\ln(b/a)}$$

# Microstrip



(a)



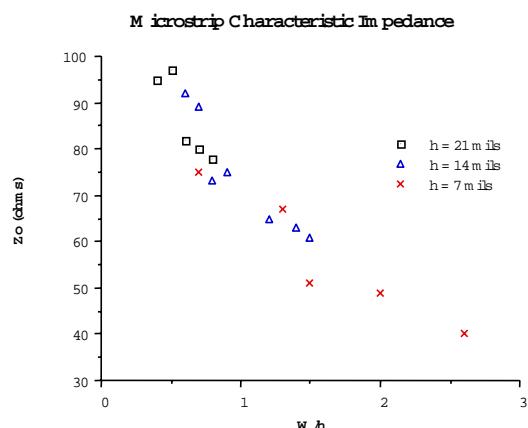
(b)

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# Microstrip



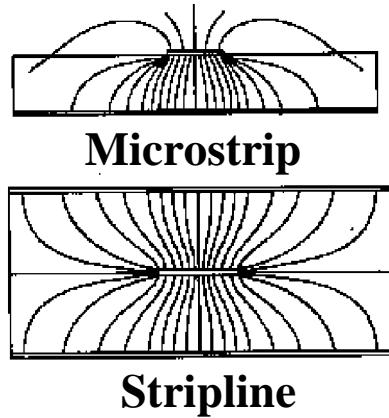
dielectric constant : 4.3.

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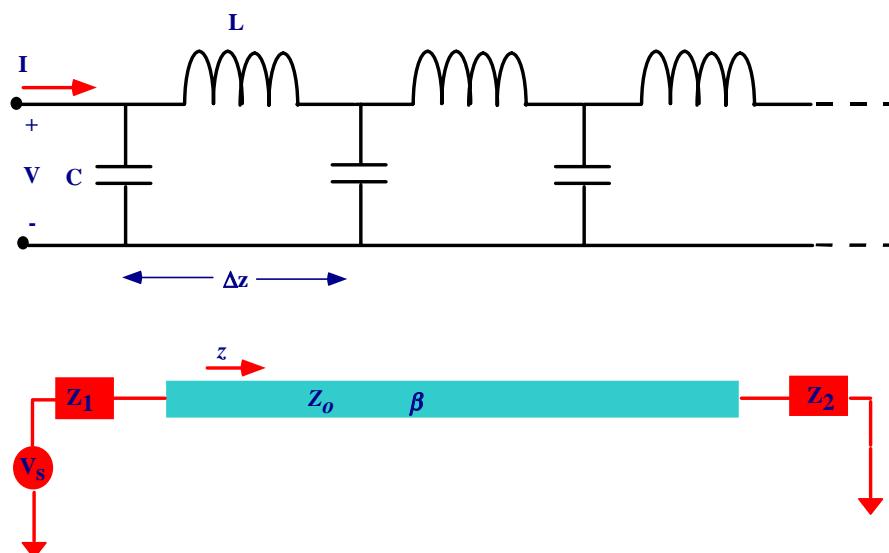
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# Electric Field Configuration

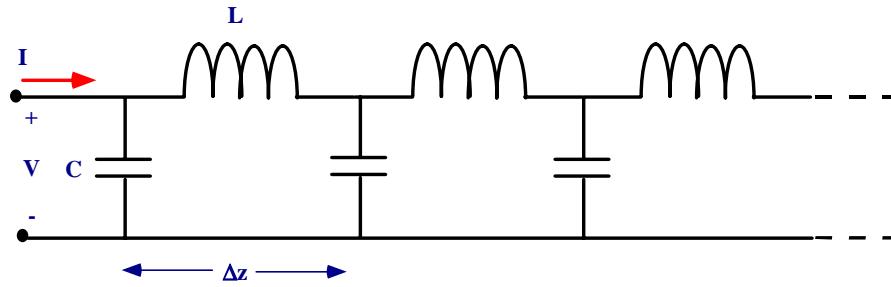


**Consequence:** Wave propagation in stripline is closer to the TEM mode of propagation and the propagation of velocity is approximately  $c/\sqrt{\epsilon_r}$ .

## TEM PROPAGATION



# Telegrapher's Equations



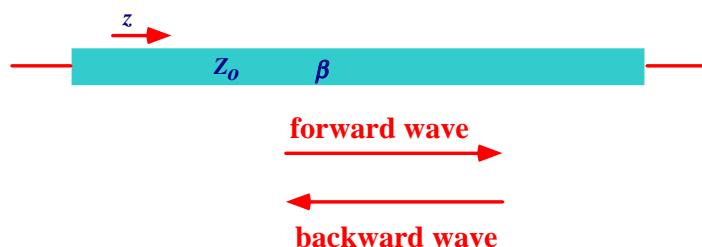
$$-\frac{\partial V}{\partial z} = L \frac{\partial I}{\partial t}$$

$$-\frac{\partial I}{\partial z} = C \frac{\partial V}{\partial t}$$

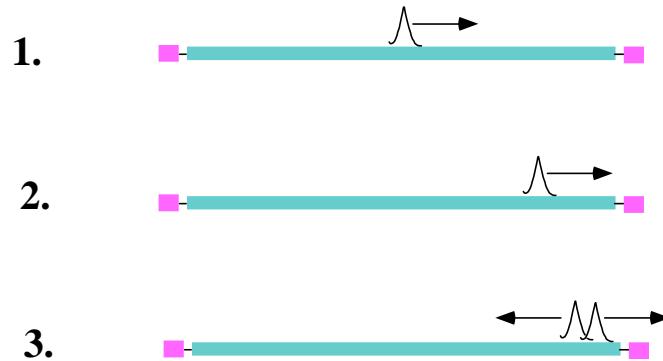
**L:** Inductance per unit length.

**C:** Capacitance per unit length.

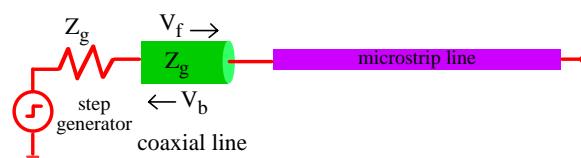
## Transmission Line Solutions



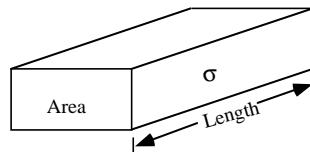
# Reflection in Transmission Lines



## Time Domain Reflectometry



# Metallic Conductors



Resistance : R

$$R = \frac{\text{Length}}{\sigma \text{ Area}}$$

Package level:

W=3 mils

R=0.0045 Ω/mm

Submicron level:

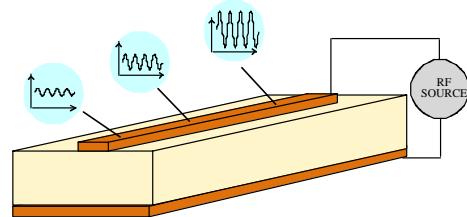
W=0.25 microns

R=422 Ω/mm

## Metallic Conductors

Metal	$\sigma$ ( $\Omega^{-1} m^{-1} \times 10^{-7}$ )
Silver	6.1
Copper	5.8
Gold	3.5
Aluminum	1.8
Tungsten	1.8
Brass	1.5
Solder	0.7

# Loss in Transmission Lines

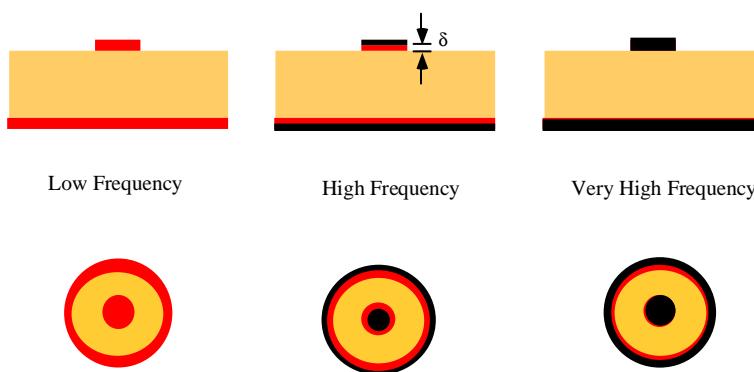


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# Skin Effect in Transmission Lines

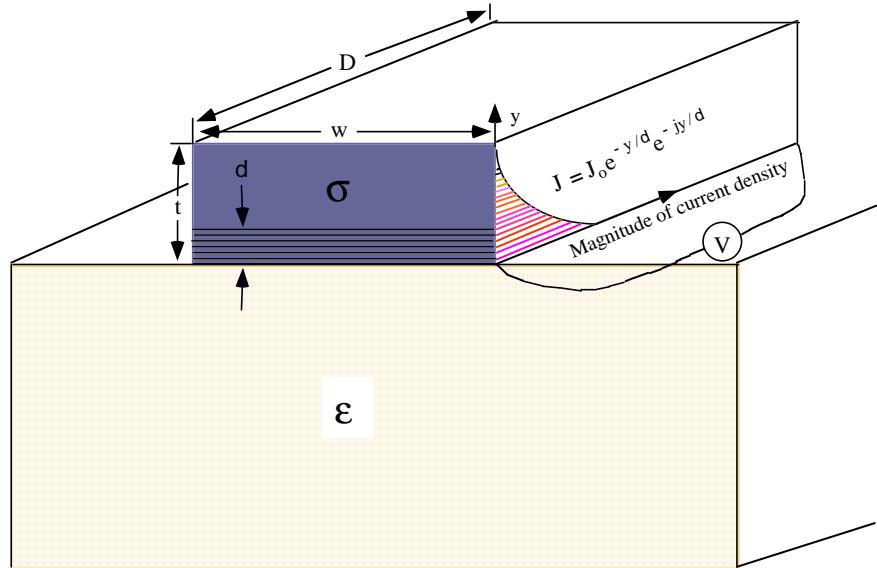


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# Skin Effect in Microstrip



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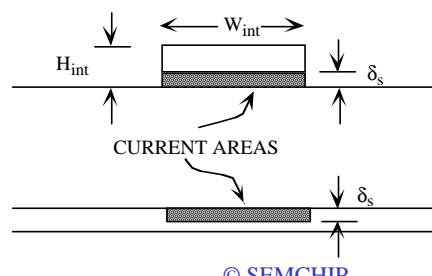
## Skin Effect

**The electric field in a material medium propagates as**

$$\text{Wavy line} \quad E_0 e^{-\gamma z} = E_0 e^{-\alpha z} e^{-j\beta z}$$

where  $\gamma = \alpha + j\beta$ . We also have

$$\gamma = \omega \sqrt{\mu \epsilon (1 + j \frac{\sigma}{\omega \epsilon})}.$$

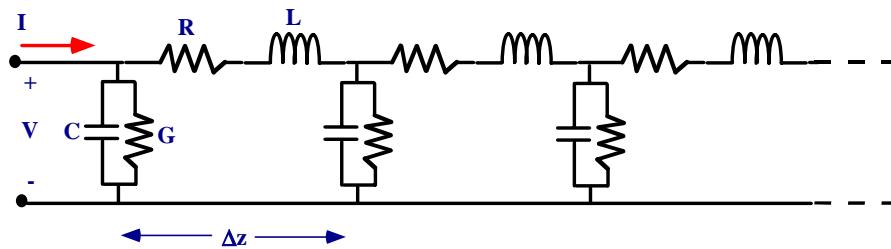


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# Lossy Transmission Line

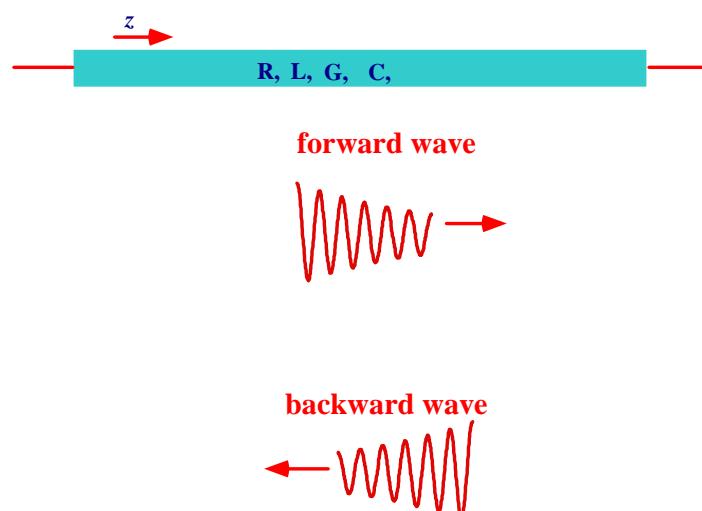


## Telegraphers Equation

$$-\frac{\partial V}{\partial z} = (R + j\omega L)I = ZI$$

$$-\frac{\partial I}{\partial z} = (G + j\omega C)V = YV$$

# Lossy Transmission Line



# Network Analyzer

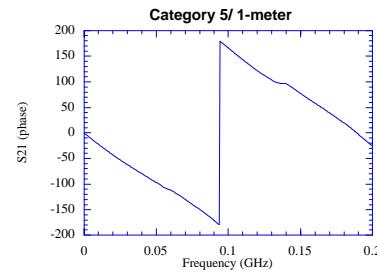
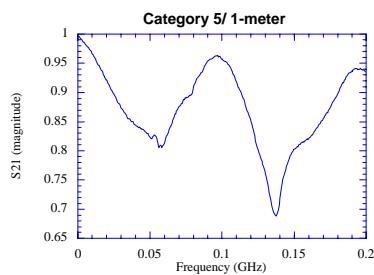
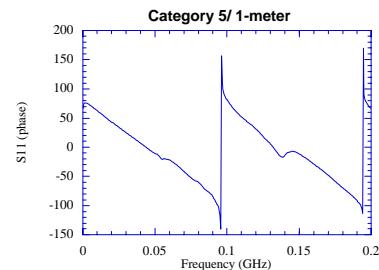
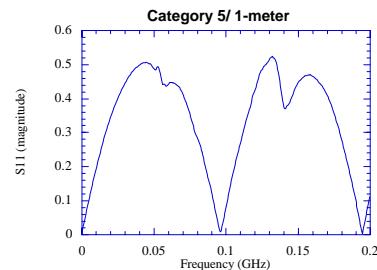


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## S Parameters of Transmission Lines Short line

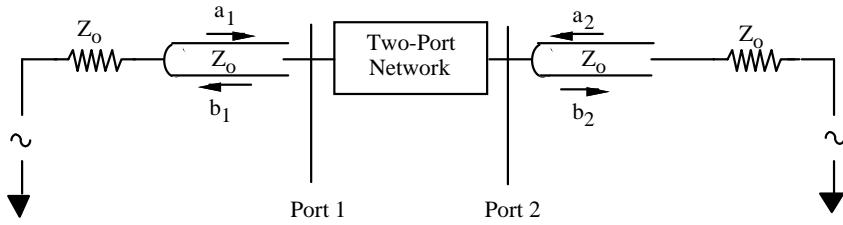


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## Two-Port Characterization



$$b_1 = S_{11} a_1 + S_{12} a_2$$

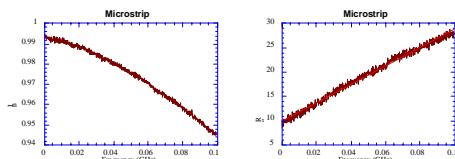
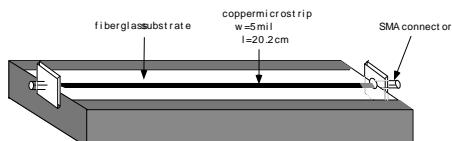
$$b_2 = S_{21} a_1 + S_{22} a_2$$

$$S_{11} = \frac{b_1}{a_1|_{a_2=0}} \quad S_{21} = \frac{b_2}{a_1|_{a_2=0}}$$

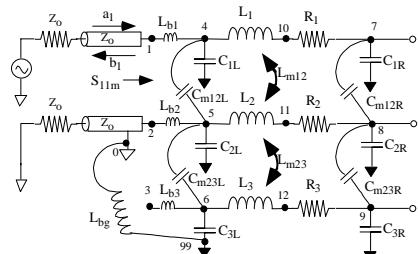
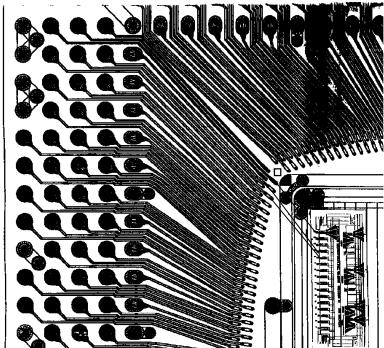
$$S_{12} = \frac{b_1}{a_2|_{a_1=0}} \quad S_{22} = \frac{b_2}{a_2|_{a_1=0}}$$

## Microstrip Characterization

- Network analyzer measurement of S parameters
- Use de-embedding scheme
- Use extraction algorithm



# Package Level Characterization



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# Crosstalk Noise and Coupled Transmission Lines

*José Schutt-Ainé  
SemChip*

## DAC Tutorial

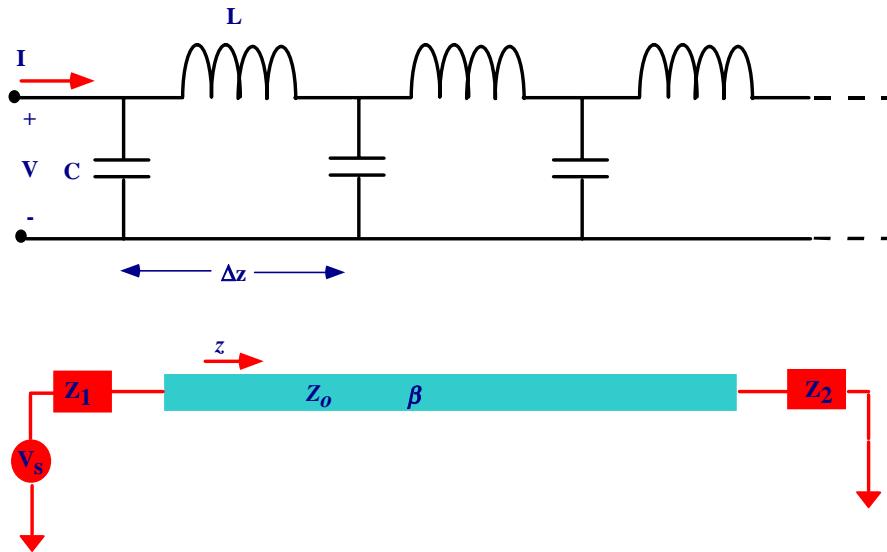
22, June 2001

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# TEM PROPAGATION

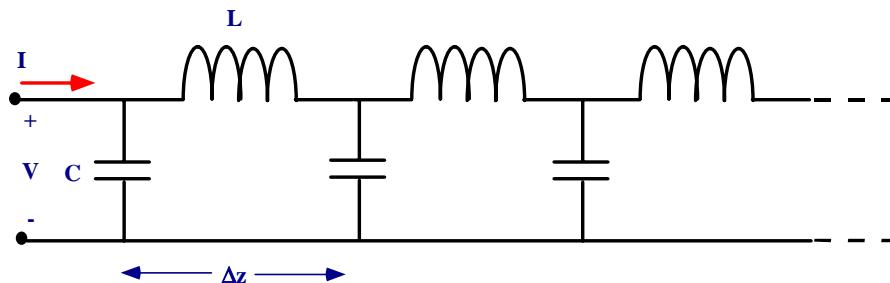


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## Telegrapher's Equations



$$-\frac{\partial V}{\partial z} = L \frac{\partial I}{\partial t}$$

$$-\frac{\partial I}{\partial z} = C \frac{\partial V}{\partial t}$$

**L:** Inductance per unit length.

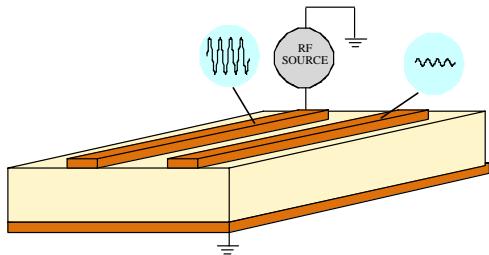
**C:** Capacitance per unit length.

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# Crosstalk and Coupled Line Analysis



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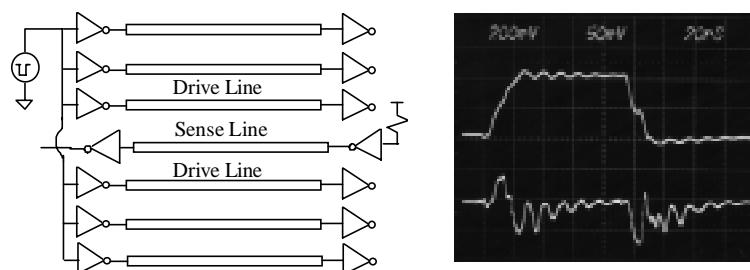
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## Crosstalk Noise

### Signal Integrity

Crosstalk	Dispersion	Attenuation
Reflection	Distortion	Loss
Delta I Noise	Ground Bounce	Radiation

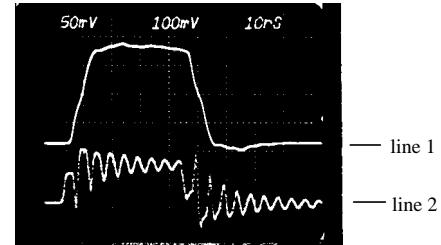
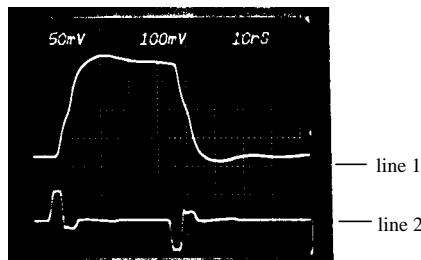
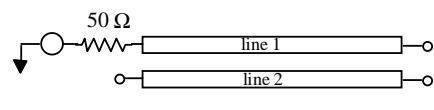
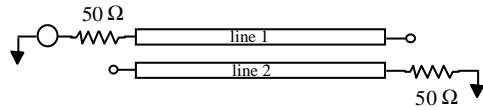


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## Crosstalk noise depends on termination

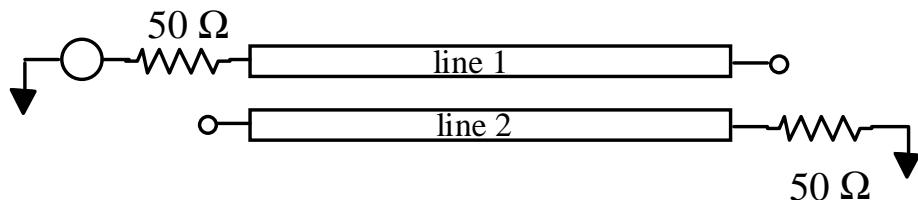


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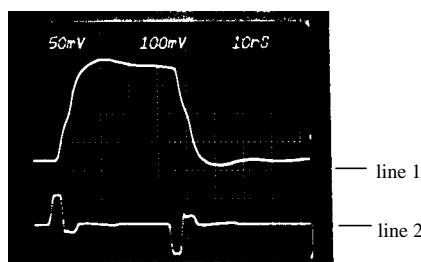
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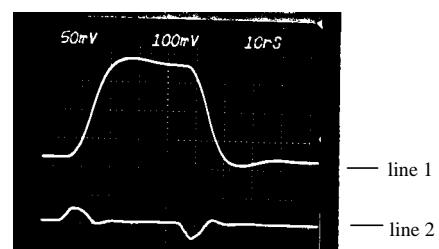
## Crosstalk depends on signal rise time



$$t_r = 1\ \text{ns}$$



$$t_r = 7\ \text{ns}$$

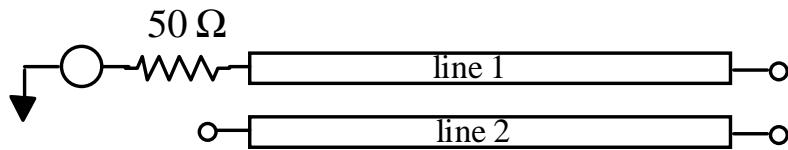


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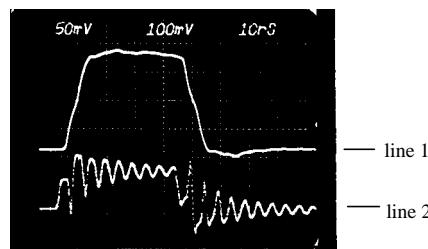
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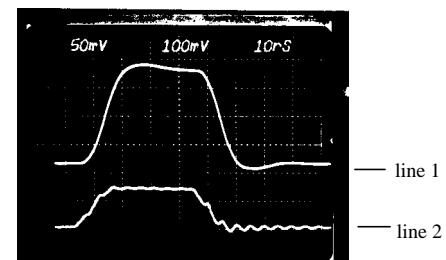
## Crosstalk depends on signal rise time



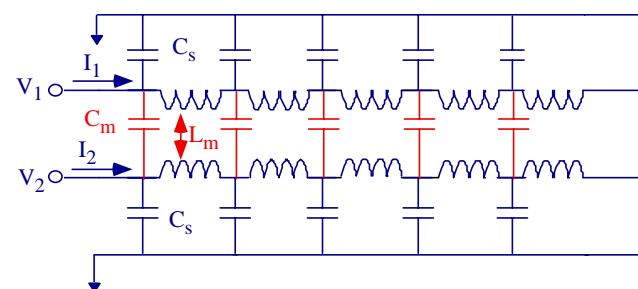
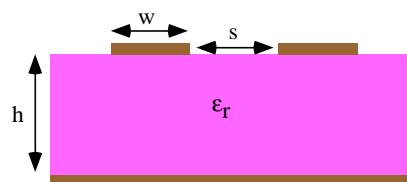
$t_r = 1 \text{ ns}$



$t_r = 7 \text{ ns}$



## Coupled Transmission Lines



## Telegraphers Equations for Coupled Transmission Lines

### Maxwellian Form

$$-\frac{\partial V_1}{\partial z} = L_{11} \frac{\partial I_1}{\partial t} + L_{12} \frac{\partial I_2}{\partial t}$$

$$-\frac{\partial V_2}{\partial z} = L_{21} \frac{\partial I_1}{\partial t} + L_{22} \frac{\partial I_2}{\partial t}$$

$$-\frac{\partial I_1}{\partial z} = C_{11} \frac{\partial V_1}{\partial t} + C_{12} \frac{\partial V_2}{\partial t}$$

$$-\frac{\partial I_2}{\partial z} = C_{21} \frac{\partial V_1}{\partial t} + C_{22} \frac{\partial V_2}{\partial t}$$

## Telegraphers Equations for Coupled Transmission Lines

### Physical form

$$-\frac{\partial V_1}{\partial z} = L_s \frac{\partial I_1}{\partial t} + L_m \frac{\partial I_2}{\partial t}$$

$$-\frac{\partial V_2}{\partial z} = L_m \frac{\partial I_1}{\partial t} + L_s \frac{\partial I_2}{\partial t}$$

$$-\frac{\partial I_1}{\partial z} = C_s \frac{\partial V_1}{\partial t} + C_m \frac{\partial V_1}{\partial t} - C_m \frac{\partial V_2}{\partial t}$$

$$-\frac{\partial I_2}{\partial z} = -C_m \frac{\partial V_1}{\partial t} + C_m \frac{\partial V_2}{\partial t} + C_s \frac{\partial V_2}{\partial t}$$

# Relations Between Physical and Maxwellian Parameters

$$L_{11} = L_{22} = L_s$$

$$L_{12} = L_{21} = L_m$$

$$C_{11} = C_{22} = C_s + C_m$$

$$C_{12} = C_{21} = -C_m$$

## Even Mode

$$\begin{aligned}-\frac{\partial V_e}{\partial z} &= (L_{11} + L_{12}) \frac{\partial I_e}{\partial t} \\-\frac{\partial I_e}{\partial z} &= (C_{11} + C_{12}) \frac{\partial V_e}{\partial t}\end{aligned}$$

Add voltage and current equations

$$V_e : \text{Even mode voltage} \quad V_e = \frac{1}{2}(V_1 + V_2)$$

$$I_e : \text{Even mode current} \quad I_e = \frac{1}{2}(I_1 + I_2)$$

$$Z_e = \sqrt{\frac{L_{11} + L_{12}}{C_{11} + C_{12}}} = \sqrt{\frac{L_s + L_m}{C_s}}$$

Impedance

$$v_e = \frac{1}{\sqrt{(L_{11} + L_{12})(C_{11} + C_{12})}} = \frac{1}{\sqrt{(L_s + L_m)C_s}}$$

velocity

## Odd Mode

$$-\frac{\partial V_d}{\partial z} = (L_{11} - L_{12}) \frac{\partial I_d}{\partial t} \quad \text{Subtract voltage and current equations}$$

$$-\frac{\partial I_d}{\partial z} = (C_{11} - C_{12}) \frac{\partial I_d}{\partial t}$$

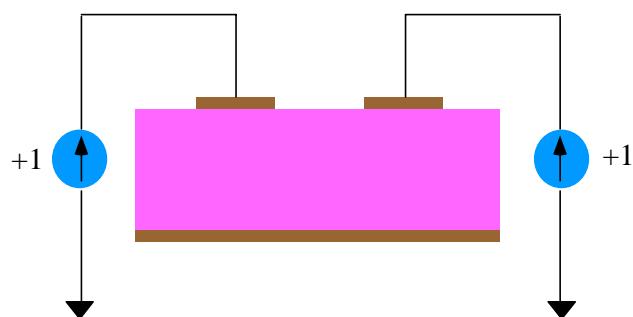
**V<sub>d</sub>** : Odd mode voltage     $V_d = \frac{1}{2}(V_1 - V_2)$

**I<sub>d</sub>** : Odd mode current     $I_d = \frac{1}{2}(I_1 - I_2)$

$$Z_d = \sqrt{\frac{L_{11} - L_{12}}{C_{11} - C_{12}}} = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}} \quad \text{Impedance}$$

$$v_d = \frac{1}{\sqrt{(L_{11} - L_{12})(C_{11} - C_{12})}} = \frac{1}{\sqrt{(L_s - L_m)(C_s + 2C_m)}} \quad \text{velocity}$$

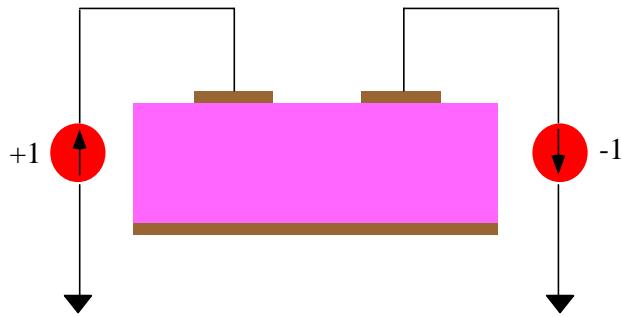
## Even Mode



$$Z_e = \sqrt{\frac{L_s + L_m}{C_s}} \quad \text{Impedance}$$

$$v_e = \frac{1}{\sqrt{(L_s + L_m)C_s}} \quad \text{velocity}$$

## Odd Mode



$$Z_d = \sqrt{\frac{L_s - L_m}{C_s + 2C_m}}$$

**Impedance**

$$v_d = \frac{I}{\sqrt{(L_s - L_m)(C_s + 2C_m)}}$$

**velocity**

## PHYSICAL SIGNIFICANCE OF EVEN- AND ODD-MODE IMPEDANCES

- \*  $Z_e$  and  $Z_d$  are the wave resistance seen by the even and odd mode travelling signals respectively.
- \* The impedance of each line is no longer described by a single characteristic impedance; instead, we have

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

## EVEN AND ODD-MODE IMPEDANCES

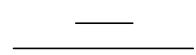
**Z<sub>11</sub>, Z<sub>22</sub> : Self Impedances**

**Z<sub>12</sub>, Z<sub>21</sub> : Mutual Impedances**

**For symmetrical lines,**

**Z<sub>11</sub> = Z<sub>22</sub> and Z<sub>12</sub> = Z<sub>21</sub>**

### EXAMPLE (Microstrip)



$$\epsilon_r = 4.3$$
$$Z_s = 56.4 \Omega$$



$$\epsilon_r = 4.3$$

$$Z_e = 68.1 \Omega \quad Z_d = 40.8 \Omega$$
$$Z_{11} = 54.4 \Omega \quad Z_{12} = 13.6 \Omega$$

Single Line

Dielectric height = 6 mils

Width = 8 mils

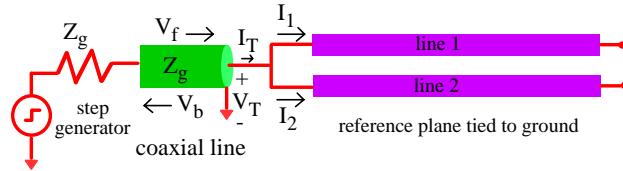
Coupled Lines

Height = 6 mils

Width = 8 mils

Spacing = 12 mils

## Even Mode

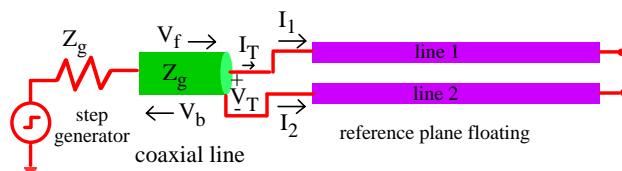


$$I_{tdr} = \left[ \frac{a_e(t,0)}{Z_e} + \frac{a_d(t,0)}{Z_d} \right] + \left[ \frac{a_e(t,0)}{Z_e} - \frac{a_d(t,0)}{Z_d} \right]$$

$$V_{tdr} = a_e(t,0) - a_d(t,0) \quad a_d(t,0) = 0$$

$$\frac{V_{tdr}}{I_{tdr}} = \frac{Z_e}{2} \quad Z_e = 2 \left( \frac{1 + \rho_e}{1 - \rho_e} \right) Z_g \quad v_e = \frac{2l}{\tau_e}$$

## Odd Mode



$$V_{tdr} = a_e(t,0) + a_d(t,0) - [a_e(t,0) - a_d(t,0)] = V_f + V_b$$

$$I_{tdr} = \left[ \frac{a_e(t,0)}{Z_e} + \frac{a_d(t,0)}{Z_d} \right] \quad I_{tdr} = - \left[ \frac{a_e(t,0)}{Z_e} - \frac{a_d(t,0)}{Z_d} \right]$$

$$a_e(t,0) = 0, \quad \frac{V_{tdr}}{I_{tdr}} = 2Z_d$$

$$Z_d = \frac{1}{2} \left( \frac{1 + \rho_d}{1 - \rho_d} \right) Z_g, \quad v_d = \frac{2l}{\tau_d}$$

## EXTRACT INDUCTANCE AND CAPACITANCE COEFFICIENTS

$$L_s = \frac{1}{2} \left[ \frac{Z_e}{v_e} + \frac{Z_d}{v_d} \right]$$

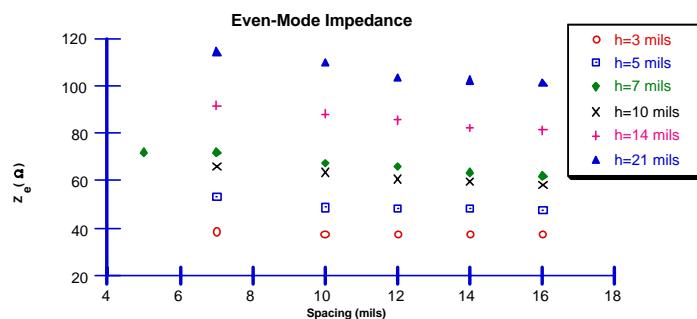
$$C_s = \frac{1}{Z_e v_e}$$

$$L_m = \frac{1}{2} \left[ \frac{Z_e}{v_e} - \frac{Z_d}{v_d} \right]$$

$$C_m = \frac{1}{2} \left[ \frac{1}{Z_e v_e} - \frac{1}{Z_d v_d} \right]$$

$$Z_d < Z_s < Z_e$$

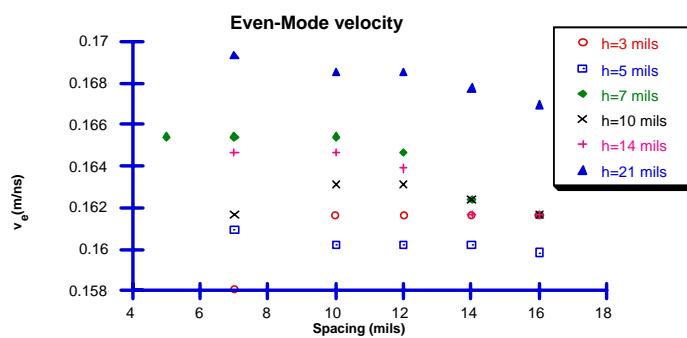
## Measured even-mode impedance



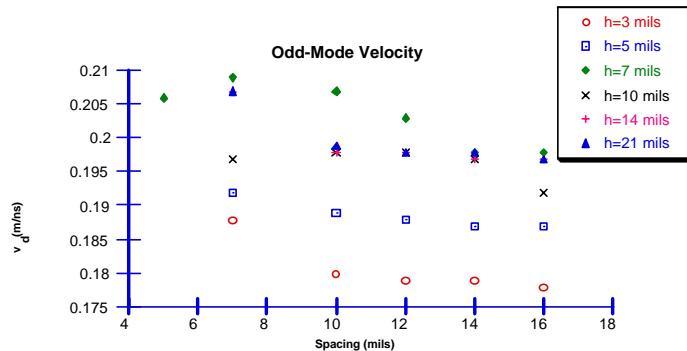
## Measured odd-mode impedance



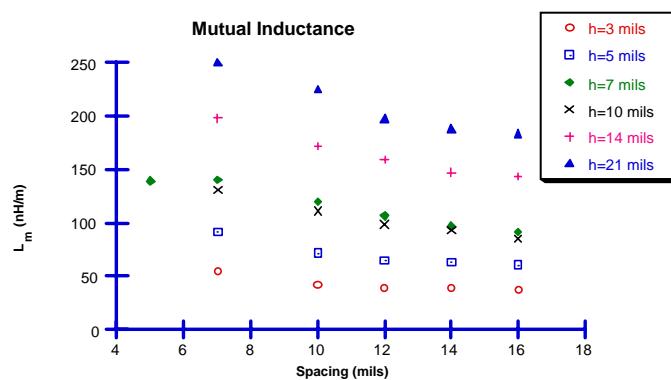
## Measured even-mode velocity



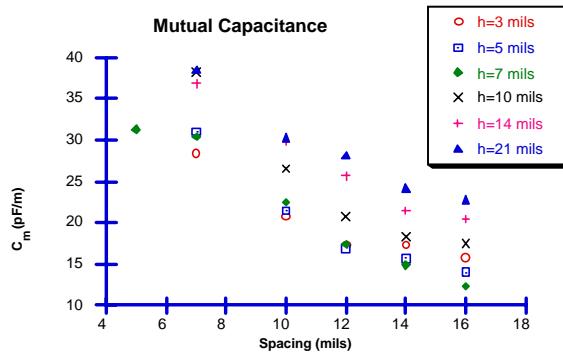
## Measured odd-mode velocity



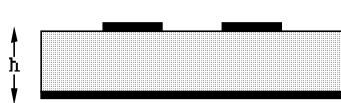
## Measured mutual inductance



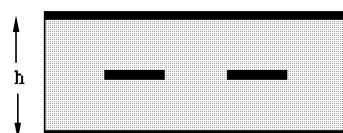
## Measured mutual capacitance



## Modal Velocities in Stripline and Microstrip

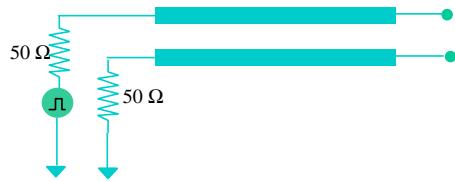


Microstrip : Inhomogeneous structure, odd and even-mode velocities must have different values.



Stripline : Homogeneous configuration, odd and even-mode velocities have approximately the same values.

## Microstrip vs Stripline



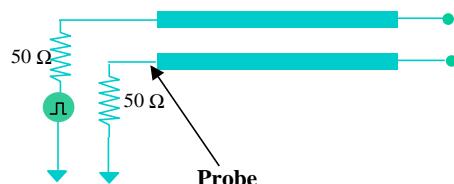
### Microstrip (h = 8 mils)

w = 8 mils  
 $\epsilon_r = 4.32$   
 $L_s = 377 \text{ nH/m}$   
 $C_s = 82 \text{ pF/m}$   
 $L_m = 131 \text{ nH/m}$   
 $C_m = 23 \text{ pF/m}$   
 $v_e = 0.155 \text{ m/ns}$   
 $v_d = 0.178 \text{ m/ns}$

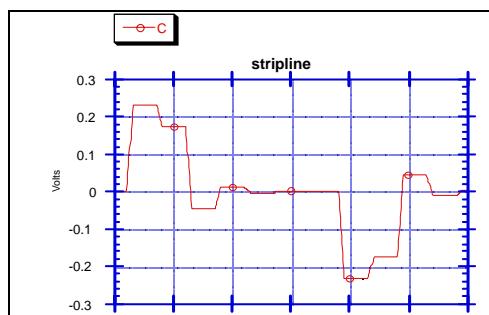
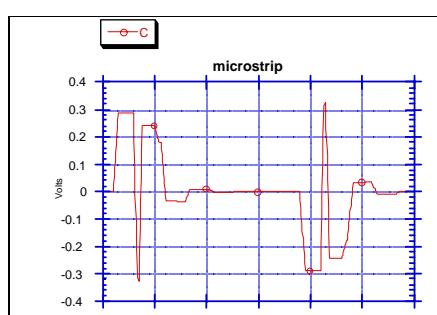
### Stripline (h = 16 mils)

w = 8 mils  
 $\epsilon_r = 4.32$   
 $L_s = 466 \text{ nH/m}$   
 $C_s = 86 \text{ pF/m}$   
 $L_m = 109 \text{ nH/m}$   
 $C_m = 26 \text{ pF/m}$   
 $v_e = 0.142 \text{ m/ns}$   
 $v_d = 0.142 \text{ m/ns}$

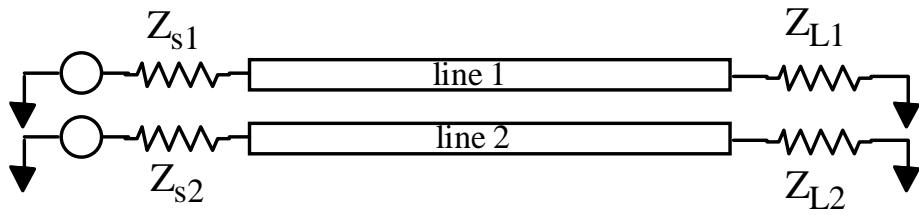
## Microstrip vs Stripline



### Sense line response at near end



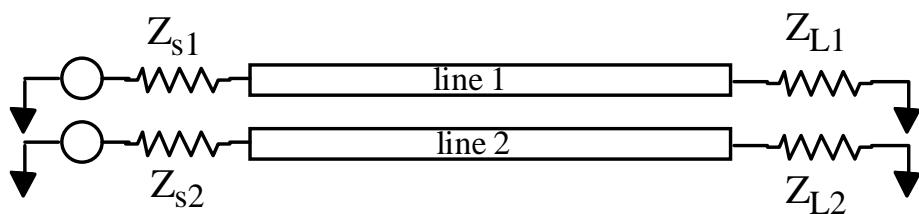
## General Solution for Voltages



$$V_1(z) = \underbrace{A_e e^{-\frac{j\omega z}{v_e}} + B_e e^{\frac{j\omega z}{v_e}}}_{even} + \underbrace{A_d e^{-\frac{j\omega z}{v_d}} + B_d e^{\frac{j\omega z}{v_d}}}_{odd}$$

$$V_2(z) = \underbrace{A_e e^{-\frac{j\omega z}{v_e}} + B_e e^{\frac{j\omega z}{v_e}}}_{even} - \underbrace{A_d e^{-\frac{j\omega z}{v_d}} - B_d e^{\frac{j\omega z}{v_d}}}_{odd}$$

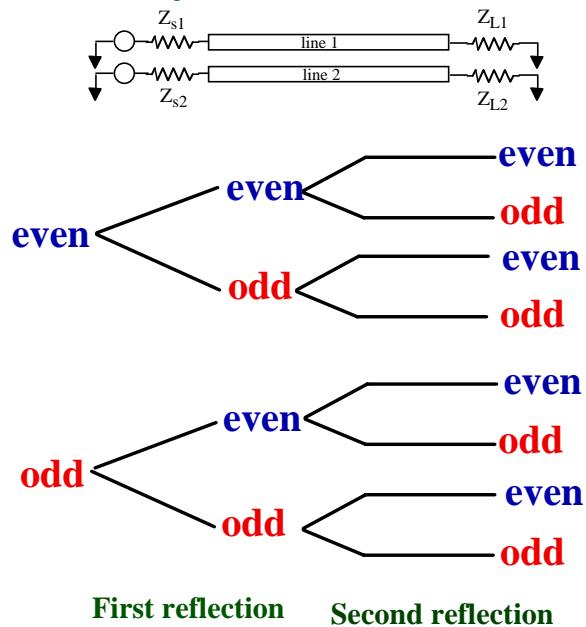
## General Solution for Currents



$$I_1(z) = \frac{1}{Z_e} \left[ \underbrace{A_e e^{-\frac{j\omega z}{v_e}} - B_e e^{\frac{j\omega z}{v_e}}}_{even} \right] + \frac{1}{Z_d} \left[ \underbrace{A_d e^{-\frac{j\omega z}{v_d}} - B_d e^{\frac{j\omega z}{v_d}}}_{odd} \right]$$

$$I_2(z) = \frac{1}{Z_e} \left[ \underbrace{A_e e^{-\frac{j\omega z}{v_e}} - B_e e^{\frac{j\omega z}{v_e}}}_{even} \right] - \frac{1}{Z_d} \left[ \underbrace{A_d e^{-\frac{j\omega z}{v_d}} - B_d e^{\frac{j\omega z}{v_d}}}_{odd} \right]$$

## Coupling of Modes (asymmetric load)

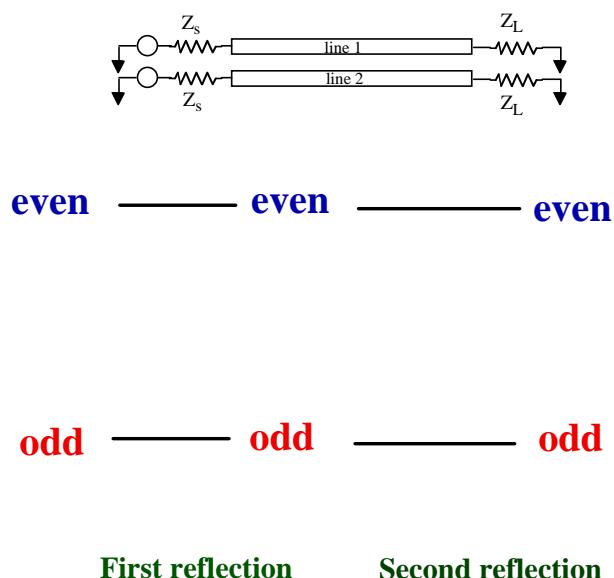


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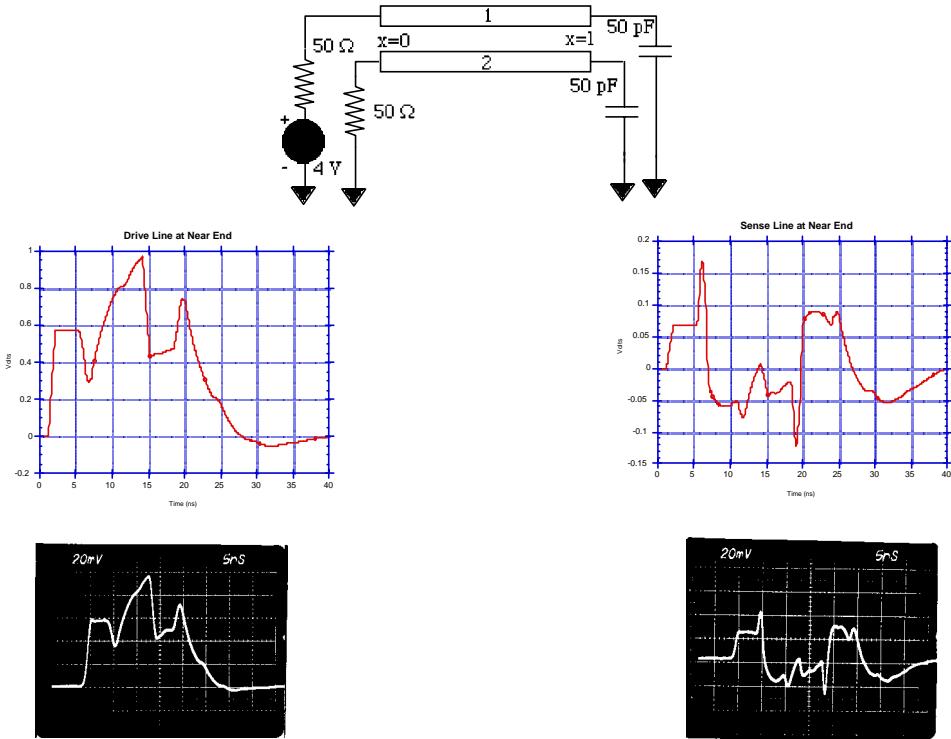
## Coupling of Modes (symmetric load)



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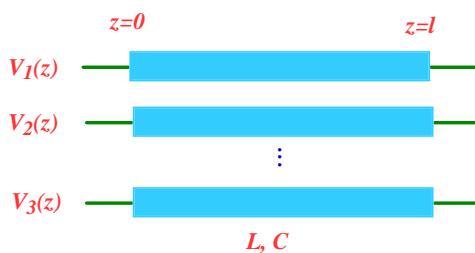


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## TELEGRAPHER'S EQUATION FOR N COUPLED TRANSMISSION LINES



$$-\frac{\partial V}{\partial z} = L \frac{\partial I}{\partial t}$$

$$-\frac{\partial I}{\partial z} = C \frac{\partial V}{\partial t}$$

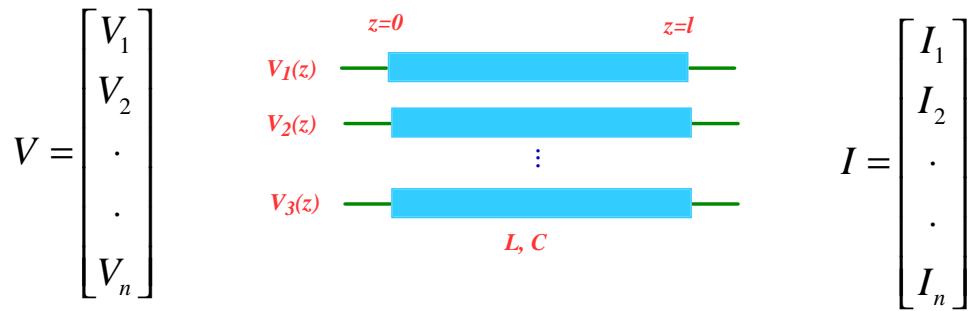
$V$  and  $I$  are the line voltage and line current VECTORS respectively (dimension  $n$ ).

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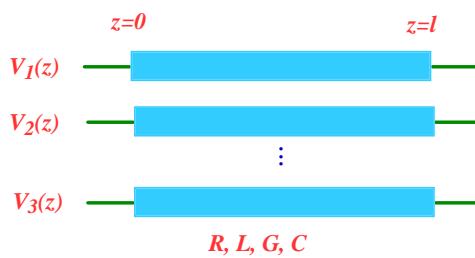
## N-LINE SYSTEM



$L$  and  $C$  are the inductance and capacitance MATRICES respectively

$$L = \begin{bmatrix} L_{11} & L_{12} & \cdot & \cdot \\ L_{21} & L_{22} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & L_{nn} \end{bmatrix} \quad C = \begin{bmatrix} C_{11} & C_{12} & \cdot & \cdot \\ C_{21} & C_{22} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & C_{nn} \end{bmatrix}$$

## COUPLED LOSSY TRANSMISSION LINES

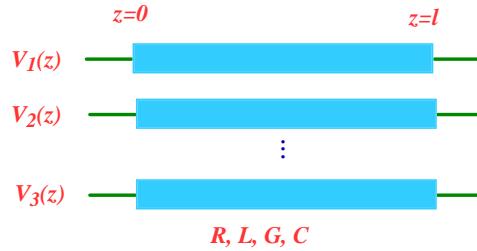


## Time Domain

$$-\frac{\partial V}{\partial z} = RI + L \frac{\partial I}{\partial t}$$

$$-\frac{\partial I}{\partial z} = GV + C \frac{\partial V}{\partial t}$$

# COUPLED LOSSY TRANSMISSION LINES



## Frequency Domain

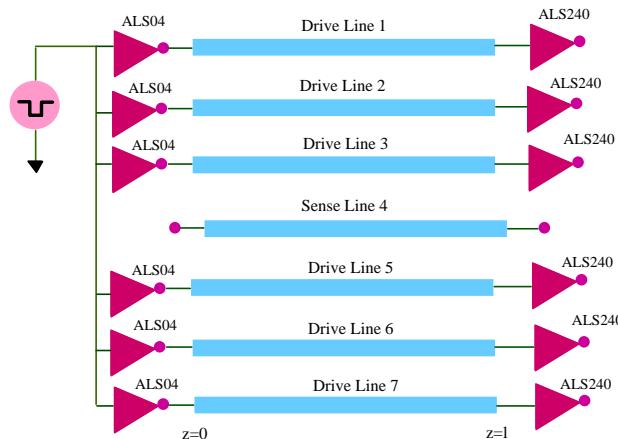
$$-\frac{\partial V}{\partial z} = ZI$$

$$-\frac{\partial I}{\partial z} = YV$$

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

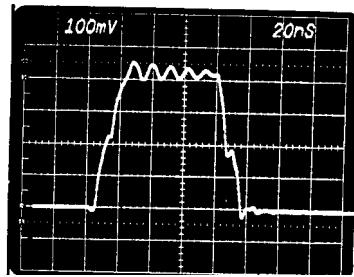
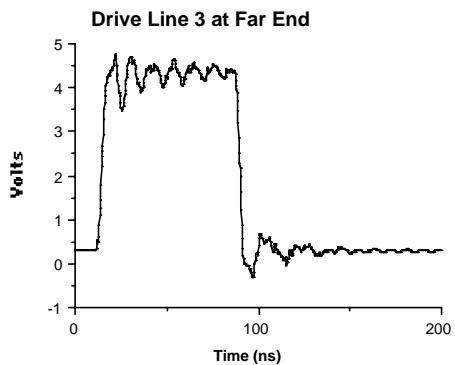
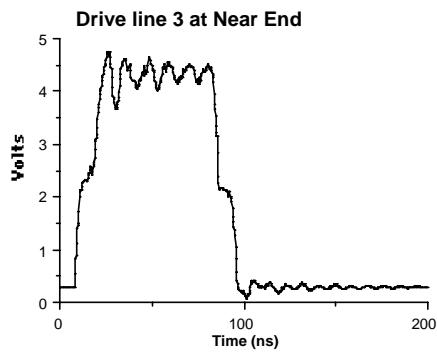
## 7-Line Coupled-Microstrip System



$$L_s = 312 \text{ nH/m}; \quad C_s = 100 \text{ pF/m};$$

$$L_m = 85 \text{ nH/m}; \quad C_m = 12 \text{ pF/m}.$$

## Drive Line 3

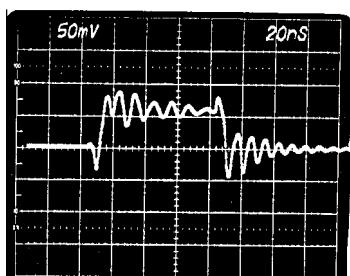
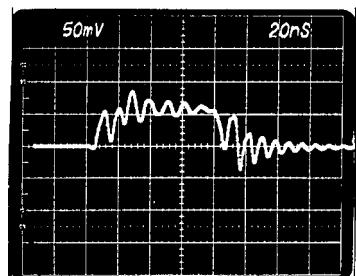
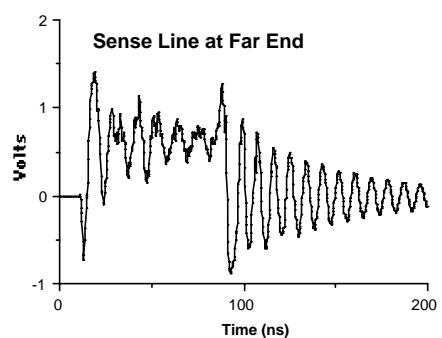
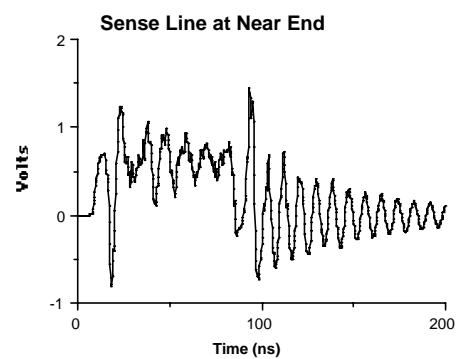


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## Sense Line



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