Indian Institute of Science Education and Research, Trivandrum

PHY-321 **Stat-Mech problem set 2** **22.1.2015**

Note: Deadline: 30 January 2015 (12:00 noon)

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1. **Gaussian Distribution**

   The binomial distribution of probabilities is given by

   $$P_n = {}^N C_n \, p^n \, q^{N-n} \tag{1}$$

   is the binomial coefficient giving the number of distinct ways of sorting N objects into two piles containing n and Nn respectively. Use the Stirling's approximation to show that

   $$s(n) \equiv \ln P_n \simeq n \ln p + (N-n) \ln(1-p) + N \ln N - (N-n) ln(N-n) - n \ln n \qquad N, n \gg 1 \tag{2}$$

   Show that

   $$\begin{aligned} s(n) &= \ln p - \ln(1-p) + \ln(N-n) \ln n \\ s(n) &= \frac{N}{n(N-n)} \end{aligned} \tag{3}$$

   Now make a Taylor expansion to second order in $x = n - Np$ to give:

   $$s(x) \simeq s(Np) - \frac{x^2}{2N(1-p)p} \tag{4}$$

   Why is there no linear term in x?

   Thus deduce

   $$P_n \simeq P_{Np} \exp\left(-\frac{(n-Np)2}{2N(1-p)p}\right)$$

   By considering the next term in the Taylor expansion show that that Gaussian distribution is a good approximation to the binomial if $|n - \overline{n}| \ll N^{2/3}$.

2. **Single particle and many particles difference**

   Consider a system of N particles, each of which can exist in either of two states, of energies 0 and ϵ. Suppose that each particle is equally likely to be in either of its two states. Write down expressions for: (a) the mean energy of one particle, and its standard deviation; (b) the mean energy of the system, and its standard deviation. Comment.

3. **Ideal gas in a box**

   Consider a cubic box of side $L = 1m$ containing $N = 6 \times 10^{23}$ ideal gas molecules. Estimate the probability of finding all the molecules in the left hand half of the box. How long might you expect to wait before observing one of the special arrangements whose probability you have just estimated. [Help: Under typical conditions it might take a molecule of the order of $10^3\,s$ to diffuse from one half of the box to the other.]

4. **Magnet**

   Let us consider an array of N magnetic atoms which we will refer to as the 'model magnet'. There are N atoms with magnetic dipole moments $\mathbf{m}$ in a magnetic field $\mathbf{H}$. The energy of a dipole is given by the interaction with the field

   $$\epsilon = -\mathbf{m} \cdot \mathbf{H}$$

   Like the two state system in quantum mechanics, the dipole moment $\mathbf{m}$ is either parallel to $\mathbf{H}$ and $\epsilon = -m\,H$ or antiparallel to $\mathbf{H}$ and $\epsilon = m\,H$.

   For the case $N = 4$ identify explicitly, and count, the different microstates associated with each possible energy macrostate (identified by each possible value of the system energy E).