EP 222: Classical Mechanics
Tutorial Sheet 3

This tutorial sheet contains problems related to the central force problem.

1. Suppose a satellite is moving around a planet in a circular orbit of radius r_0. Due to a collision with another object, satellite's orbit gets perturbed. Show that the radial position of the satellite will execute simple harmonic motion with $\omega = \frac{l}{mr_0^2}$, where l is the initial angular momentum of the satellite.

2. A particle of mass m is moving under the influence of a central force $\mathbf{F}(\mathbf{r}) = -\frac{C}{r^3}\hat{\mathbf{r}}$, with $C > 0$. Find the nonzero values of angular momentum l for which the particle will move in a circular orbit.

3. In the lectures, we obtained the equation of the orbit (an equation connecting r and θ) for the Kepler's problem ($V(r) = -k/r$), by solving a second order differential equation for the variable $u = 1/r$. Show that one gets the same result if one directly integrates the integral connecting the r and θ coordinates, derived in the lectures.

4. Two particles move about each other in circular orbits under the influence of gravitational forces, with a period τ. Their motion is suddenly stopped at a given instant of time, and they are then released and allowed to fall into each other. Prove that they collide after a time $\tau/4\sqrt{2}$.

5. Show that the motion of a particle in the potential field

$$V(r) = -\frac{k}{r} + \frac{h}{r^2}$$

is the same as that of the motion under the Kepler potential alone when expressed in terms of a coordinate system rotating or precessing around the center of force. For negative total energy, show that if the additional potential term is very small compared to the Kepler potential, then the angular speed of precession of the elliptical orbit is

$$\dot{\Omega} = \frac{2\pi mh}{l^2\tau}.$$

The perihelion of Mercury is observed to precess (after correction for known planetary perturbations) at the rate of $40''$ of arc per century. Show that this precession could be accounted for classically if the dimensionless quantity

$$\eta = \frac{h}{ka}$$

(which is a measure of the perturbing inverse-square potential relative to the gravitational potential) were as small as 7×10^{-8}. (The eccentricity of Mercury's orbit is 0.206, and its period is 0.24 year.)

6. A geostationary orbit is one in which a satellite moves in a circular orbit at the given height in the equatorial plane, so that its angular velocity of rotation around earth is same as earth's angular velocity, thereby, making it look stationary when seen from a point on equator. Assuming that the earth's rotational velocity, and radius, respectively, are $\Omega_e = \frac{2\pi}{86400}$ rad/s, and$R_e = 6400$ km, calculate the altitude of the satellite, and its orbital velocity.

7. A space company wants to launch a satellite of mass $m = 2000$ kg, in an elliptical orbit around earth, so that the altitude of the satellite above earth at perigee is 1100 kms, and at apogee it is 35,850 kms. Assuming that the launch takes place at the equator, calculate: (a) energy of the satellite in the elliptical orbit, (b) energy required to launch the satellite, (c) eccentricity of the orbit, (d) angular momentum of the satellite, and (e) speeds of the satellite at apogee and perigee. Use the values of R_e and Ω_e specified in the previous problem.

8. The ultimate aim of the space company of the previous problem is to put the satellite in a geostationary orbit. Therefore, after launching it in the elliptical orbit, the company wants to transfer it in a geostationary orbit by firing rockets at the apogee to increase its speed to the required one. How much change in speed is needed to put the satellite in the geostationary orbit, and how much energy will be required to achieve that change?

9. Examine the scattering produced by a repulsive central force $f = kr^{-3}$. Show that the differential cross section is given by

$$\sigma(\Theta)d\Theta = \frac{k}{2E}\frac{(1-x)dx}{x^2(2-x)^2 \sin \pi x},$$

where $x = \Theta/\pi$, and E is energy.