

## EP 222: Classical Mechanics Tutorial Sheet 4

This tutorial sheet contains problems related to rigid body kinematics.

1. (a) By examining the eigenvalues of an antisymmetric  $3 \times 3$  real matrix  $A$ , show that  $I \pm A$  is nonsingular, where  $I$  is the identity matrix.  
(b) Show then that under the same conditions the matrix

$$B = (I + A)(I - A)^{-1}$$

is orthogonal.

2. Show that the components of the angular velocity along the space set of axes are given in terms of the Euler angles by

$$\begin{aligned}\omega_x &= \dot{\theta} \cos \phi + \dot{\psi} \sin \theta \sin \phi, \\ \omega_y &= \dot{\theta} \sin \phi - \dot{\psi} \sin \theta \cos \phi \\ \omega_z &= \dot{\psi} \cos \theta + \dot{\phi}\end{aligned}$$

3. A particle is thrown up vertically with initial speed  $v_0$ , reaches a maximum height and falls back to ground. Show that the Coriolis deflection when it again reaches the ground is opposite in direction, and four times greater in magnitude, than the Coriolis deflection when it is dropped at rest from the same maximum height.
4. A projectile is fired horizontally along Earth's surface. Show that to a first approximation the angular deviation from the direction of fire resulting from the Coriolis effect varies linearly with time at a rate  $\omega \cos \theta$ , where  $\omega$  is the angular frequency of Earth's rotation and  $\theta$  is co-latitude, the direction of deviation being to the right in the northern hemisphere.
5. A wagon wheel with spokes is mounted on a vertical axis so it is free to rotate in the horizontal plane. The wheel is rotating with an angular speed of  $\omega = 3.0$  radians/s. A bug crawls out on one of the spokes of the wheel with a velocity of  $0.5$  cm/s holding on the spoke with a coefficient of friction  $\mu = 0.30$ . How far can the bug crawl along the spoke before it starts to slip?