PH 422: Quantum Mechanics II
Tutorial Sheet 1

This tutorial sheet contains problems related to the addition of angular momenta for quantum mechanical particles.

1. Verify the values of the following C-G coefficients

   (a) $\langle j1j0|j1jj\rangle = \sqrt{\frac{j}{j+1}}$

   (b) $\langle j2j0|j2jj\rangle = \sqrt{\frac{j(2j-1)}{(j+1)(2j+3)}}$

2. Compute the following C-G coefficients

   (a) $\langle j_1, 1/2, m-1/2, 1/2|j_1 1/2, j_1 \pm 1/2, m\rangle = \pm\sqrt{\frac{j_1 \pm m + 1/2}{2j_1+1}}$

   (b) $\langle j_1, 1/2, m+1/2, -1/2|j_1, 1/2, j_1 \pm 1/2, m\rangle = \sqrt{\frac{j_1 \mp m + 1/2}{2j_1+1}}$

3. Show that the eigenvectors of total angular momentum $\mathbf{J}$, obtained by coupling the orbital angular momentum (l) and the spin angular momentum ($s = 1/2$) of an electron can be written as

$$\mathcal{Y}_l^{jm} = \mathcal{Y}_l^{l\pm 1/2, m} = \frac{1}{\sqrt{2l+1}} \begin{pmatrix} \pm\sqrt{l \pm m + \frac{1}{2}}Y_l^{m-1/2}(\theta, \phi) \\ \sqrt{l \mp m + \frac{1}{2}}Y_l^{m+1/2}(\theta, \phi) \end{pmatrix}.$$

   Also verify that $\mathcal{Y}_l^{jm}$ is an eigenfunction of $\mathbf{J}^2$ operator, where $\mathbf{J} = \mathbf{L} + \mathbf{S}$.

4. Suppose you have two spin $\frac{1}{2}$ particles, with their individual spin operators $\mathbf{S_1}$ and $\mathbf{S}_2$. Obtain the eigenstates of $\mathbf{S^2}$ and $\mathbf{S}_z$ operators, where $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$, by the following two approaches:

   (a) Using the C-G coefficients

   (b) By constructing the $\mathbf{S}^2$ operator in the uncoupled basis, and diagonalizing it.

5. Calculate the C-G coefficients needed to couple the two angular momenta $j_1 = 3/2$ and $j_2 = 1$ to the possible j values, and express the coupled states $|j_1 j_2 jm\rangle$ in terms of the uncoupled state $|j_1 j_2 m_1 m_2\rangle$.