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PH 422 L 1 / Slide 1

QM I → Basic principles

Harmonic Oscill.

Hydrogen atom

Angular momentum

QM II Perturbation theory

Time dependent phenomena

Scattering theory

Quantum Mechanics

Recap of principles :

1. Physical states in QM obey Principle of Linear Superposition

$$|\psi\rangle = \frac{c_1|\psi_1\rangle + c_2|\psi_2\rangle}{\sqrt{|c_1|^2 + |c_2^2|}}$$

1. The set of all possible states forms a "vector space"

Vector space → basis

2. Dynamical variables are linear operators acting on this vector space

$$A|\psi\rangle = |\psi_A\rangle$$

$$A\left(c_1|\psi_1\rangle + c_2|\psi_2\rangle\right)$$
$$= c_1\,A|\psi_1\rangle + c_2\,A|\psi_2\rangle$$

2a. Observables represented by hermitian operators

. . . . real eigenvalues

2b. Symmetry operations represented by unitary operators

3. Probability interpretation

Observable : energy

Set of all eigenstates form a basis

$$|\psi\rangle = \sum_{\text{all } n} c_n |E_n\rangle$$

- Observation returns only one eigenvalue
- $|c_n|^2$ represents the probability of finding eigenvalue E_n.

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4. Quantum "kinematics"

The commutator algebra of the operators needs to be specified

Canonical formalism:

Express all dynamical variables as functions of (q_i, p_i) $i = 1, \dots N$

For the <u>phase space</u> variables q_i, p_i we propose

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$$[q_i , p_j] \equiv q_i p_j - p_j q_i$$

$$= i\hbar \{q_i , p_j\}$$

PB → Poisson bracket

$$= i\hbar \delta_{ij}$$

Commutator algebra

1. $[A, B] = -[B, A]$

2. $[c_1 A_1 + c_2 A_2 , B] = c_1 [A_1 , B] + c_2 [A_2 , B]$

3. $[A,[B,C]] + [C,[A,B]] + [B,[C,A]] = 0$

Note also, $[q_i, q_j] = [p_i, p_j] = 0$

5. Dynamics

$i\hbar \dot{O} = [O, H]$

.... Heisenberg picture

States indep. of time

Schrödinger picture $|\psi t\rangle$

$i\hbar \frac{\partial}{\partial t} |\psi t\rangle = H |\psi t\rangle$

$\dot{p} = -\frac{\partial V}{\partial q}$

$= -\frac{\partial H}{\partial q}$

$= -\{H, p\}_{PB}$

"Wave function" notation

$$\underbrace{\langle q}|\psi\, t\rangle \equiv \psi(q,t)$$

↳ basis labelled by eigenvalues of position operator

Or, in momentum basis $|p\rangle$,

$$\varphi_p(t) \equiv \langle p|\psi\, t\rangle$$

Or, $\varphi_n(t) = \langle E_n|\psi\, t\rangle$ ...energy basis

6. Identical quanta

→ BE / FD statistics

"Quanta are not particles" !!

	Cl. Mech.	BE	FD
H H	1/4	1/3	0
H T	} 1/2	1/3	1
T H			
T T	1/4	1/3	0