Since sin matches with growing exponential, this part of ψ_{II} must be zero, i.e.

$$\cos\left(\int_b^a k(x')\,dx'\right) = 0$$

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i.e. $$\int_b^a k(x')\,dx' = \left(n+\tfrac{1}{2}\right)\pi$$

$$\boxed{n = 0, 1, 2 \ldots}$$

i.e. $$\boxed{\int_b^a \sqrt{\frac{2m}{\hbar^2}\left(E_n - V(x)\right)}\,dx = \left(n+\tfrac{1}{2}\right)\pi}$$

This has to be read as a highly implicit equation for possible binding energies E_n ....

... where one must remember that E dependence exists in the classical turning points a & b as well

The quantum number n designates energies. To lowest approx. we expect H.O. like energies near the local minimum.

More general observation : n can be interpreted as the number of nodes (zeros) of the wave function. ..

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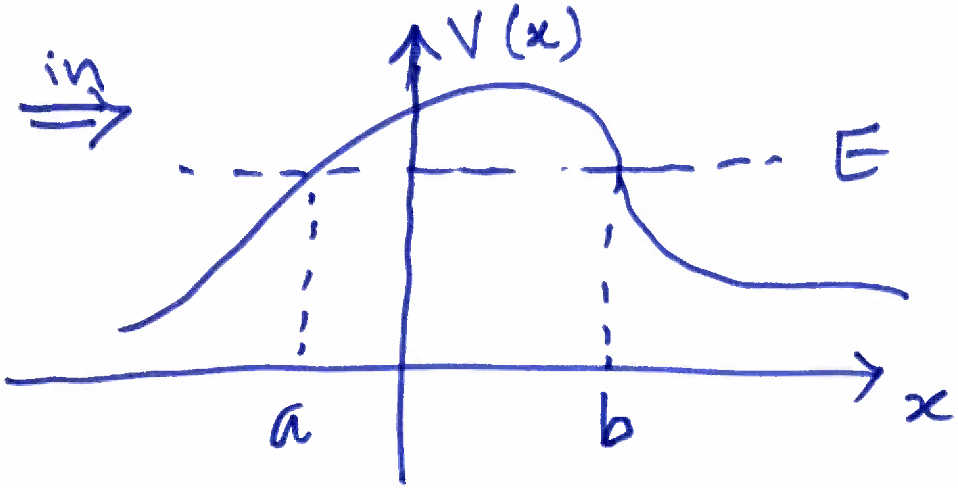
Note also the analogy to "old Quantum theory" generalising Bohr quantization condition as

" $\oint p\, dq = nh$ "

... Sommerfeld formula ... since $p = \sqrt{2m(E - V(x))}$ and a factor 2 explaining the "$\oint$" covering the closed classical trajectory

$b \rightarrow a \rightarrow b$

Application to barrier penetration



Transmission coeff.

$$T = \exp\left(-2\int_a^b \chi(x)\,dx\right)$$

$$\boxed{T = \exp\left(-2\int_a^b \sqrt{\frac{2m}{\hbar^2}\left(V(x)-E\right)}\,dx\right)}$$

|| 2 from squaring the amplitude

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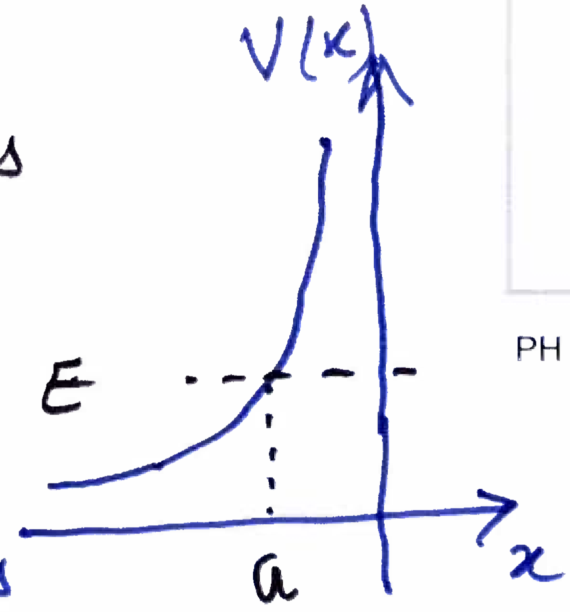
Example

α particles as projectiles hitting Gold nuclei



Although classically, reaching the core nucleus of Gold ~~would~~ would require prohibitively large energy $\rightarrow \dfrac{(Ze)(2e)}{(10 f)}$

However due to tunnelling, the α particle may penetrate the Coulomb barrier and then undergo a nuclear reaction

Thus, here $K = \frac{\sqrt{2mE}}{\hbar}\left(\frac{a}{x}-1\right)^{1/2}$

Thus the factor in the exponent of the formula for T is

$$\int_a^0 K(x)\,dx = \sqrt{2mE}\int_a^0 \sqrt{\frac{a}{x}-1}\;dx$$

$$= \frac{Z \times 2}{\hbar v}\, e^2 \pi$$

$$v = \sqrt{2E/m}$$

$\frac{2Ze^2}{a} = E$ determines a

$E \rightarrow$ initial KE of α particle

Radioactive decay