

# Time dependent perturbation theory

( cdeep.iitb.ac.in )

It is possible to split (as before)

$$H = H_0 + V$$

... but  $V$  is  $V(t)$ , time dependent

(i) Explicit  $t$ -dependence  $\rightarrow$  (a) transient for a short time (b) oscillatory/periodic time dep.

(ii) Situation stationary;  $V$  is actually time indep, but system sees  $V$  for a limited time  $\rightarrow$  "scattering"



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## Interaction picture:

A hybrid between Schrödinger & Heisenberg pictures. Pioneered by Dirac



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Idea: We know: for  $H_0$ , let

$$H_0 |n\rangle = E_n |n\rangle$$

$H_0 \rightarrow$  no explicit  
 $t$ -dep.

Then for a state  $|d\rangle$   
in Schrödinger picture,

$$|d, t=0\rangle = \sum_n c_n |n\rangle$$

$$\text{Then } |d, t\rangle_S = \sum_n c_n e^{-iE_n t/\hbar} |n\rangle$$

idea of interaction picture:  
 build in the time dependence due  
 to  $V(t)$  in the coeff.s  $c_n$ :

$$|\alpha t\rangle_I \sim \sum_n c_n(t) e^{-iE_n t/\hbar} |n\rangle$$

Check: for  $H_0$  case:

$$|\alpha t=0\rangle_S = \sum_n c_n |n\rangle$$

$$c_n = \langle n | \alpha t=0 \rangle_S \dots \text{consider making } c_n \rightarrow c_n(t)$$

$$i\hbar \frac{\partial}{\partial t} |\alpha t\rangle_S = i\hbar \frac{\partial}{\partial t} \sum_n c_n^{(0)}(t) |n\rangle$$

$$\text{should} = H_0 |\alpha t\rangle_S = \sum_n c_n^{(0)}(t) E_n |n\rangle$$



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$$\text{i.e. } i\hbar \frac{d}{dt} C_n^{(0)}(t) = E_n C_n^{(0)}(t)$$

$$\therefore C_n^{(0)} = C_n^{(0)}(t=0) e^{-i E_n t / \hbar}$$



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Thus propose interaction picture:

$$|a, t\rangle_I = e^{iH_0 t / \hbar} |a, t\rangle_S \quad \dots \text{with the split } H = H_0 + V(t)$$

Compare  $|a, t\rangle_H = e^{iHt/\hbar} |a, t\rangle_S$

& for operator  $A$ ,  $A_H(t) = e^{iHt/\hbar} A_S e^{-iHt/\hbar}$

$$A_I(t) = e^{iH_0 t / \hbar} A_S e^{-iH_0 t / \hbar}$$

Consider diff. eqn. for  $|d\rangle_I$ :

$$i\hbar \frac{\partial}{\partial t} |d t\rangle_I = i\hbar \frac{\partial}{\partial t} (e^{iH_0 t/\hbar} |d t\rangle_S)$$

$$= \underbrace{-H_0 e^{iH_0 t/\hbar}} |d t\rangle_S + e^{iH_0 t/\hbar} \underbrace{(H_0 + V)} |d t\rangle_S$$

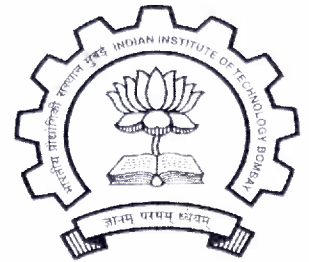
$$= e^{iH_0 t/\hbar} \underbrace{V e^{-iH_0 t/\hbar} e^{iH_0 t/\hbar}}_{\text{insert}} |d t\rangle_S$$

$$= \underbrace{V_I(t)}_{\text{regroup}} |d t\rangle_I$$

We can now see that this matches the

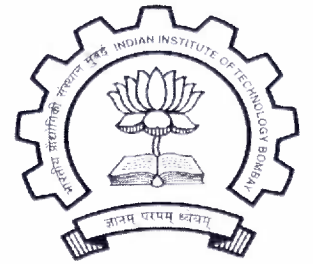
expectation we had,  $|d t\rangle_I \sim \sum C_n(t) e^{-iE_n t/\hbar} |n\rangle$

and we try to check,  $i\hbar \frac{\partial C_n}{\partial t} \sim V_I C_n$



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$$\text{Let } C_n(t) = \langle n | \alpha(t) \rangle_I$$

$$i\hbar \frac{\partial}{\partial t} C_n(t) = \langle n | i\hbar \frac{\partial}{\partial t} |\alpha(t)\rangle_I$$

$$= \langle n | V_I(t) |\alpha(t)\rangle_I$$

$$\underbrace{\sum_m |m\rangle \langle m|}$$

$$= \sum_m \langle n | V_I(t) |m\rangle C_m(t)$$

$$\text{Now } \langle n | e^{iH_0 t/\hbar} V_S e^{-iH_0 t/\hbar} |m\rangle = e^{i(E_n - E_m)t/\hbar} \langle n | V_S |m\rangle$$
$$\equiv e^{i\omega_{nm} t} V_{nm}$$

$$\omega_{nm} \equiv (E_n - E_m)/\hbar$$

Thus

$$i\hbar \frac{\partial}{\partial t} C_n(t) = \sum_m e^{i\omega_{mn}t} V_{nm} C_m(t)$$

... typically any one level (say  $n=0$  i.e. ground state) may be occupied to begin with, and above equation tells how other levels  $k^m$  connected to  $n$  by matrix elements  $V_{nm}$  begin to get occupied.



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