Time dependent perturbation

Exact treatment of 2 level system

example

$$H_0 = E_1 |1\rangle\langle 1| + E_2 |2\rangle\langle 2|$$

$$V(t) = \gamma e^{i\omega t} |1\rangle\langle 2| + \gamma e^{-i\omega t} |2\rangle\langle 1|$$

$$H = \begin{pmatrix} E_1 & \gamma e^{i\omega t} \\ \gamma e^{-i\omega t} & E_2 \end{pmatrix}$$

$$V_{11} = V_{22} = 0$$

Then in interaction picture,

$$i\hbar \frac{d}{dt} c_1(t) = e^{i\omega_{21} t} \gamma e^{i\omega t} c_2(t)$$

$$i\hbar \frac{d}{dt} c_2(t) = e^{i\omega_{12} t} \gamma e^{-i\omega t} c_1(t)$$

$$= e^{-i\omega_{21} t - i\omega t} \gamma c_1(t)$$

Separate the eqn. for c_2

$$i\hbar \ddot{c}_2 = -i(\omega_{21}+\omega)\gamma e^{-i(\omega_{21}+\omega)t} c_1$$

$$+ \gamma e^{-i(\omega_{21}+\omega)t} \frac{1}{i\hbar} e^{i(\omega_{21}+\omega)t} c_2$$

$$= \frac{\gamma^2}{i\hbar} c_2 + \cancel{\gamma}\hbar \cancel{e^{i(\omega_{21}+\omega)t}} \dot{c}_2 (\omega_{21}+\omega)$$

$$\ddot{c}_2 = -\frac{\gamma^2}{\hbar^2} c_2 - i(\omega+\omega_{21})\dot{c}_2$$

....homog. 2^{nd} order

To solve, let $c_2 = e^{i\Omega t} c_2^{(0)}$

$$-\Omega^2 + \frac{\gamma^2}{\hbar^2} + i(\omega+\omega_{21})(i\Omega) = 0$$

$$+\Omega^2 + (\omega+\omega_{21})\Omega - \gamma^2/\hbar^2 = 0$$

$$\Omega_{\pm} = -\frac{\omega+\omega_{21}}{2} \pm \sqrt{\frac{(\omega+\omega_{21})^2}{4} + \gamma^2/\hbar^2}$$

Two solutions, C_+ , C_-

$$C_2 = A_+ e^{i\Omega_+ t} + A_- e^{i\Omega_- t}$$

Assume at $t=0$, $C_1 = 1$ $\quad C_2 = 0$

$C_2(t=0) = 0 \Rightarrow A_+ + A_- = 0$

Further, $i\hbar \dot{C}_2 = i\hbar(i)(\Omega_+ A_+ + \Omega_- A_-) = \gamma$

$A_- = -A_+$ and $A_+(\Omega_+ - \Omega_-) = -\gamma/\hbar$

$$\therefore A_+ = (-\gamma/\hbar)\Big/2\left\{\frac{(\omega+\omega_{21})^2}{4} + \gamma^2/\hbar^2\right\}^{1/2}$$

$$c_2(t) = A_+\left(e^{i\Omega_+ t} - e^{i\Omega_- t}\right)$$

$$= A_+ e^{-i\left(\frac{\omega+\omega_{21}}{2}\right)t} \times 2i\sin\left[\left\{\frac{(\omega+\omega_{21})^2}{4} + \frac{\gamma^2}{\hbar^2}\right\}^{1/2} t\right]$$

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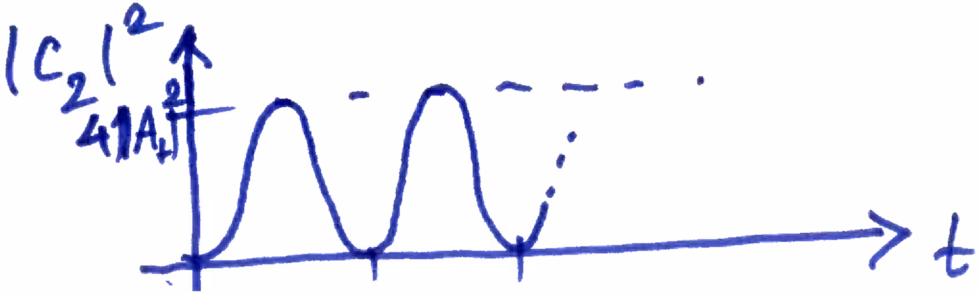
We started with only c_1 occupied.
What is the probability for c_2 to be occupied, as a function of time?

$$|c_2(t)|^2 = |A_+|^2 4\sin^2\left[\left\{\frac{(\omega+\omega_{21})^2}{4} + \frac{\gamma^2}{\hbar^2}\right\}^{1/2} t\right]$$



Note that $|c_1|^2 + |c_2|^2 = 1$

Typically $|c_2| \sim \gamma/(\omega+\omega_{21})$

$\omega = -\omega_{21} \rightarrow |c_2^{(0)}| = 1$