Photoelectric effect

↳ Using ~~constant~~ time dependent ~~pertr~~ formalism

To begin, we treat a "toy problem"

Treat a simple system subject to "harmonic perturbation"

Consider $V(t) = \mathcal{V}^{0} e^{i\omega t} + \mathcal{V}^{0\dagger} e^{-i\omega t}$

... in real situation this is the time dependence of incident "photon" field

Insert this in the iterative formula for $c_n^{(1)}$

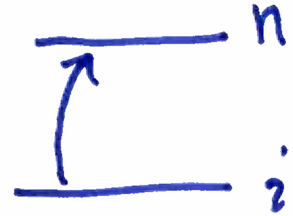
$$c_n^{(1)}(t) = \frac{-i}{\hbar}\int_0^t \left(V_{ni}\, e^{i\omega t'} + V_{ni}^{\dagger}\, e^{-i\omega t'}\right) e^{i\omega_{ni} t'}\, dt'$$

$$= \frac{1}{\hbar}\left[\frac{1-e^{i(\omega+\omega_{ni})t}}{\omega+\omega_{ni}}\, V_{ni} + \frac{1-e^{i(\omega_{ni}-\omega)t}}{-\omega+\omega_{ni}}\, V_{ni}^{\dagger}\right]$$

Two kinds of levels get excited,

$\omega_{ni} \simeq -\omega$ ... first term

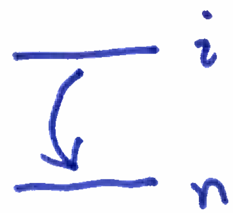
$\omega_{ni} \simeq \omega$ ... second term $\rightarrow$



absorption

Similar interpretation for first term ...

$E_n \simeq E_i - \omega$ or



emission

We can make a general observation about absorption and emission rates

First convert to a rate by using

$$\lim_{t\to\infty} \frac{1}{\omega^2} \sin^2 \frac{\omega t}{2\hbar} = \frac{\pi t}{2\hbar} \delta(\omega)$$

and applying to first (emission) & second (absorption) terms separately:

Emission, $W_{i\to\{n\}} = \frac{2\pi}{\hbar} \overline{|\mathcal{V}_{ni}|^2} \rho(E)\Big|_{E_n \simeq E_i - \hbar\omega}$

Absorption, $W_{i\to\{n\}} = \frac{2\pi}{\hbar} \overline{|\mathcal{V}^{\dagger}_{ni}|^2} \rho(E)\Big|_{E_n \simeq E_i + \hbar\omega}$

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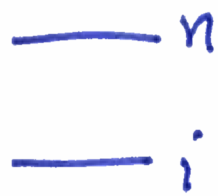
Given specific levels i & n (labelled by energy values E_i & E_n)



apply the above using the equality $|V_{ni}|^2 = |V^{\dagger}_{in}|^2$

we can deduce

$$\frac{\text{Emission rate } i \to \{n\}}{\text{density of states at } \{n\}} = \frac{\text{absorption rate } n \to \{i\}}{\text{density of states } \{i\}}$$

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Continuing from this toy solution to realistic case ....

... still treat incident light as plane waves

Classical radiation field

~~An~~ charged particle in external E-M field

$$H = \frac{1}{2m}\left|\vec{p} - \frac{e}{c}\vec{A}\right|^2 + e\phi(\vec{x},t)$$

"minimal substition"

... $\phi(\vec{x},t)$ & $\vec{A}(\vec{x},t)$ are electromagnetic scalar & vector potentials

[The resulting canonical eqn.s for $\vec{x}$ are Lorentz force law
See Goldstein chapter 1]

For QM we replace $\vec{p} = -i\hbar\vec{\nabla}$; $\cancel{H = i\hbar\frac{\partial}{\partial t}}$

Thus, expanding the above we get

$$\boxed{H \simeq \frac{1}{2m}|\vec{p}|^2 + e\phi(\vec{x},t) - \frac{e}{mc}\vec{A}\cdot\vec{p}}$$

Note,

$$(\vec{p} - \frac{e}{c}\vec{A})\cdot(\vec{p} - \frac{e}{c}\vec{A})\psi \rightarrow \left[p^2 - 2\frac{e}{c}\vec{A}\cdot\vec{p} - \frac{e}{c}(\underset{\substack{\uparrow \\ -i\hbar\vec{\nabla}}}{\vec{p}}\cdot\vec{A})\right]\psi$$

$$\cancel{+\frac{e^2}{c^2}|\vec{A}|^2\psi} \rightarrow 0$$

use gauge choice

Gauge choice $\vec{\nabla}\cdot\vec{A} = 0$, $\phi = 0$