Plane wave E-M field :

$$\vec{A} = 2A_0 \hat{\epsilon} \cos\left(\frac{\omega}{c}\hat{n}\cdot\vec{x} - \omega t\right)$$

$$= A_0 \hat{\epsilon} \left\{ \exp\left(i\frac{\omega}{c}\hat{n}\cdot\vec{x} - i\omega t\right) + \exp\left(-i\frac{\omega}{c}\hat{n}\cdot\vec{x} + i\omega t\right) \right\}$$

$e^{-i\omega t}$ part gives absorption; $e^{i\omega t}$ emission

and $V^{\dagger} = -\frac{eA_0}{mc}\left(e^{i\frac{\omega}{c}\hat{n}\cdot\vec{x}}\ \hat{\epsilon}\cdot\vec{p}\right)$

Thus, borrowing from the toy example,

$$W_{in} = \frac{2\pi}{\hbar}\frac{e^2}{m^2c^2}|A_0|^2 \Big| \underbrace{\langle n | e^{i\frac{\omega}{c}\hat{n}\cdot\vec{x}}\ \hat{\epsilon}\cdot\vec{p} | i \rangle}_{(\ldots)V_{ni}} \Big|^2 \delta(E_n - E_i - \hbar\omega)$$

for absorption

This formula is ready for calculating <u>atomic photoelectric effect</u>.

[The remaining steps are interesting example of a realistic problem in Physics]

First we change from rate → cross-section

Cross-section is defined as

$$\sigma = \frac{\text{Energy absorbed/time}}{\text{incident energy flux}} \sim [L^2]$$

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To convert $W_{i\to n}$ into σ, we need energy flux of in the E-M field

Let u = energy density in the incoming E-M wave

Then flux $= c\,u$

$$u = \frac{1}{2}\left(\frac{|\vec{E}|^2}{8\pi} + \frac{|\vec{B}|^2}{8\pi}\right)$$

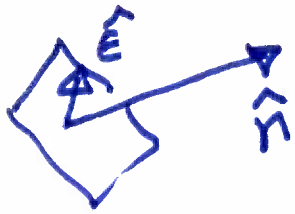
(Gaussian units for E-M)

$$\vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\vec{A} - \cancel{\vec{\nabla}\phi} \to 0$$

$$= -i\frac{\omega}{c}\vec{A}$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$= \hat{\epsilon}\times\hat{n}\; i\frac{\omega}{c}\vec{A}$$

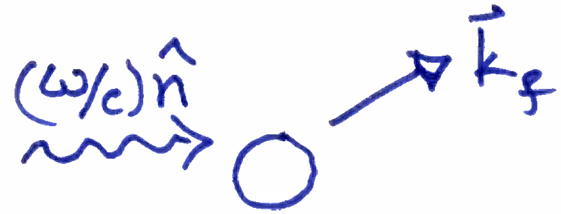


Thus $\quad c u = \frac{1}{2\pi}\frac{\omega^2}{c}|A_0|^2$

Thus,

$$\sigma_{abs} = \frac{\hbar\omega\left(\frac{2\pi}{\hbar}\right)\left(\frac{e^2}{m^2c^2}\right)|A_0|^2\left|\langle n|e^{i\frac{\omega}{c}\hat{n}\cdot\vec{x}}\,\hat{\epsilon}\cdot\vec{p}\,|i\rangle\right|^2 \cdot \delta(E_n - E_i - \hbar\omega)}{\frac{1}{2\pi}\frac{\omega^2}{c^2}|A_0|^2}$$

Density of states :



Assume emission of electrons into a final state of wave number $\vec{k}_f$

Thus we need the density of states $|\vec{k}_f\rangle$ which all have a fixed value of energy

To calculate this density ... to make the states countable, put the system in a box first,

$$\psi_{\vec{k}_f} = \langle \vec{x} | \vec{k}_f \rangle = \frac{e^{i \vec{k}_f \cdot \vec{x}}}{L^{3/2}}$$

and $\vec{k} = \frac{2\pi}{L}(n_x \hat{i} + n_y \hat{j} + n_z \hat{k})$

$n_x, n_y, n_z = 0, \pm 1, \pm 2 \ldots$

Imposing periodic boundary condition

$$\psi(\vec{x} + (\underbrace{L_x}_{} n_x \hat{i} + \underbrace{L_y}_{} n_y \hat{j} + \underbrace{L_z}_{} n_z \hat{k})) = \psi(\vec{x})$$

fixed lengths