

S-matrix and phase shifts:



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Overview:

"out"
region
space
distant
future

A diagram illustrating a localized wave packet in phase space. It features a horizontal axis labeled "time" at the bottom and a vertical axis. A circle, representing the wave packet, is centered on the time axis. Inside the circle, several diagonal hatching lines represent the wave's oscillations. Above the circle, the word "localised" is written next to a curved arrow pointing downwards. At the top of the circle, there is a small upward-pointing arrow. To the right of the circle, a curved arrow points to the left.

"in"
region

distant
past

$$H = H_0 + V$$

The picture follows QM convention

Modify & specialise the time dependent perturbation theory

Important formulae (a trailer)



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$$1. \langle \beta | S(\alpha) \rangle = \langle \beta | \alpha \rangle - \frac{i}{\hbar} \int \phi_{\beta}^*(\vec{x}'t') V(\vec{x}'t') \psi_{\alpha}^+(\vec{x}'t') d^3x' dt'$$

where \vec{x}' is supposed to be in the "out" region though formalism is more general.

ψ_{α}^+ denotes as free "in" α state already propagated correctly upto (\vec{x}', t')

2. Use of Green Function or the kernel ... to define ψ_α^+

$$\psi(x_f t_f) = i \int G_0^+(x_f t_f; x_i t_i) \psi(x_i t_i) dx_i$$

\leftarrow
use as ψ^+

$$+ \frac{1}{i} \int dt' d^3x' G_0^+(x_f t_f; x' t') V(x' t') \psi(x' t')$$

where G_0^+ is the kernel for H_0 evolution,
 "+" means propagating forward in time
 Specialise & simplify

3. Stationary collision theory

$$(H_0 - E) u_\alpha(\vec{x}) = 0 ; \quad (H - E_\alpha) \chi_\alpha(\vec{x}) = 0$$



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$$\phi_\alpha(\vec{x}, t) = e^{-i\omega_\alpha t} u_\alpha(\vec{x})$$

$$\psi^+(\vec{x}, t) = e^{-i\omega_\alpha t} \chi_\alpha^+(\vec{x})$$

Interaction picture ansatz to build in time dependence due to H_0



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4 Fermi's Golden Rule
for collision theory:

$$\langle \beta | S - 1 | \alpha \rangle = -\frac{i}{\hbar} \underbrace{\langle \beta | T | \alpha \rangle}_{\text{T-matrix}} \int_{-\infty}^{\infty} g(t) e^{i\omega_{\beta\alpha} t} dt$$

\hookrightarrow not-so-important time dep.

$$|| \quad \langle \beta | T | \alpha \rangle = \int u_\beta^*(\vec{x}') V(\vec{x}') \chi_\alpha^+(\vec{x}') d^3x'$$

$$W_{\beta \leftarrow \alpha} = \frac{2\pi}{\hbar} \delta(\beta) |\langle \beta | T | \alpha \rangle|^2$$

S-matrix : (S-operator)

This is defined as the operator

which maps eigenstates of H_0 in the

remote past to those in remote future



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$$|\beta^{\text{out}}\rangle = \sum_{\alpha} "S_{\beta\alpha} |\alpha^{\text{in}}\rangle"$$

where α, β are generic eigenstates of H_0 .

- As a caution it is assumed that the basis $\{|\beta^{\text{out}}\rangle\}$ is isomorphic to, but not identical to, the basis $\{|\alpha^{\text{in}}\rangle\}$. // The two may differ by an overall phase

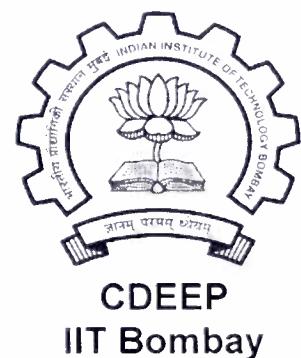
The S-matrix is unitary

$$S_{\beta\gamma}^+ S_{\gamma\lambda} = \delta_{\beta\lambda}$$

$$\underbrace{S_{\rho\sigma}}_{\text{in}} \underbrace{S_{\sigma\lambda}^+}_{\text{out}} = \delta_{\rho\lambda}$$

→ belong to same Hilbert space ; the "~~out~~"
"in"

If the rate \mathcal{W} or cross-section σ calculated
from $T = S - i\Gamma$ do not seem to obey unitarity
then there is some aspect not accounted for,
the Hilbert spaces $|out\rangle$ & $|in\rangle$ are not ~~iso~~
isomorphic \Rightarrow "error" or "new physics"



Evolution equation for the Green Function

Recall,

$$i\hbar \frac{\partial}{\partial t} |\alpha t\rangle_I = V_I(t) |\alpha t\rangle_I$$



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We propose to use $|\alpha t\rangle_I$ as the ψ^+ or x^+ states, and take them to be evolved using Green Function

$$\psi_\alpha(\vec{x}_f, t_f) = i \int G^+(\vec{x}_f, t_f; \vec{x}_i, t_i) \psi_\alpha(\vec{x}_i, t_i) d^3x_i$$

Thus, the G^+ obeys the equation

$$i\hbar \frac{\partial}{\partial t} G^+ = V_I(t) G^+ \quad \dots \text{need to clarify} "+"$$