Scattering Theory (contd)

"Stationary" scattering – time indep.

→ a steady stream of projectiles impinging on a target

Distinguished from bound state problems because of the "spetrum" (the list of all eigenvalues of an operator) of H_0 is continuum for scattering problems

Elastic scattering → all energy in the form of KE + PE ; not lost in the target or projectile

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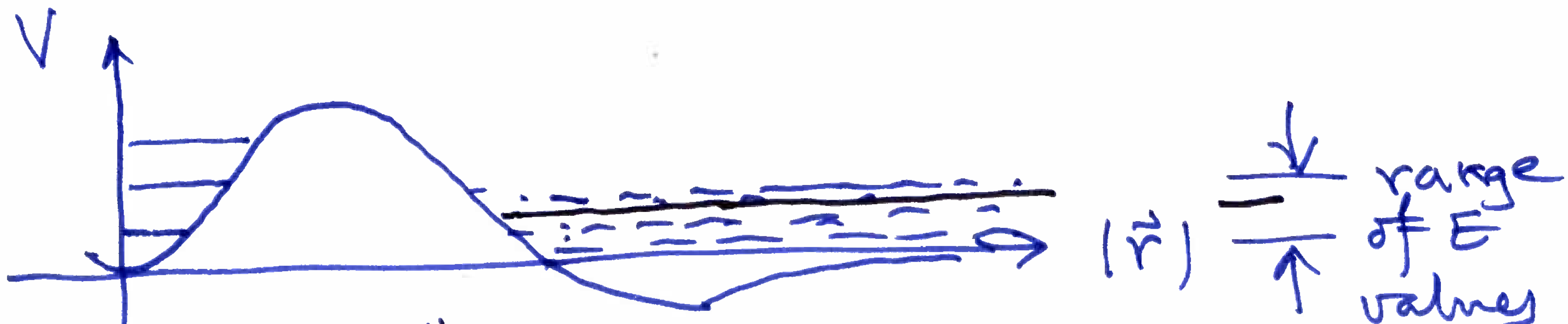
The corresponding technical/mathematical assumption is that

$$\{\text{spetrum of } H_0\} \equiv \{\text{spetrum of } H = H_0 + V\}$$

$$\text{state of } H_0 \;\not\longrightarrow\; \text{state of } H$$

but energy eigenvalue remains same

How a deviation might occur



A few "bound" states "behind" the barrier may be accessible. Then (spetrum of $H_0 + V$) $\neq$ sp. of H_0

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We can also think of G^+ in analogy with bound state problems

$$H\psi_n = (H_0 + V)(\psi_n^{(0)} + \psi_n^{(1)}) = E_n \psi_n$$

$$= E_n^{(0)}\psi_n^{(0)} + V\psi_n^{(0)} + H_0\psi_n^{(1)} + \underbrace{V\psi_n^{(1)}}_{\text{ignore}} = (E_n^{(0)} + \Delta E_n)(\psi_n^{(0)} + \psi_n^{(1)})$$

$$(H_0 - E_n^{(0)})\psi_n^{(1)} \approx -V\psi_n^{(0)}$$ (ignored some terms)

$$\psi_n^{(1)} = -(H_0 - E_n^{(0)})^{-1} V \psi_n^{(0)}$$

$$= \lim_{\epsilon \to 0} \frac{1}{E_n^{(0)} - H_0 + i\epsilon} V \psi_n^{(0)}$$

Lippmann-Schwinger Eqn.

Recall for free particles, $H_0 = \frac{|\vec{p}|^2}{2m}$

In $|\vec{p}\rangle$ basis, L-S eqn. would become

$$\varphi^{(1)}_{\vec{p}}(p) = \lim_{\epsilon \to 0} \frac{1}{\frac{|\vec{p}|^2}{2m} - H_0 + i\epsilon} V \underbrace{\varphi^{(0)}_{\vec{p}}}_{e^{i\vec{p}\cdot\vec{x}/\hbar}}$$

We solve for L-S eqn. in $\vec{x}$ basis where we think of it as solving an inhomog. diff. eqn :

$$(-H_0 + E^{(0)})\psi^{(1)}_n = V\psi^{(0)}_n \Rightarrow \frac{\hbar^2}{2m}(\nabla^2 + |\vec{k}|^2)\psi^{(1)}_n = V\psi^{(0)}_n$$

Want to recast as

$$(\nabla^2 + k^2)\, G(\vec{x} - \vec{x}') = \delta^3(\vec{x} - \vec{x}')$$

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Thus we have $(\text{Diff. op.}) f = F$

desired fun. — given forcing funct.

The solution is of the form

$$f = u + v_{\text{part. soln.}}$$

u: homog. soln.

Here the homog. soln. is the known eigenfunction
For determining particula part, set up a more general problem

$$(\text{Diff. op.}) \cancel{v_{\text{p.s.}}}\ G(\vec{x}-\vec{x}') = \delta^3(\vec{x}-\vec{x}')$$

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Above is like a "master key"

$$(\text{Diff. op})_{\vec{x}} \underbrace{\int G(\vec{x}-\vec{x}')F(\vec{x}')d^3x'}_{v_{p.s.}(\vec{x})} = \int \delta^3(\vec{x}-\vec{x}')F(\vec{x}')\, d^3\vec{x}' = F(\vec{x})$$

In Fourier transf. space it is easy to solve for

$$\tilde{G}(\vec{p}) = \int \frac{d^3(\vec{x}-\vec{x}')}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{x}-\vec{x}')} G(\vec{x}-\vec{x}')$$

$$= \int \frac{d^3\vec{x}}{(2\pi)^3} e^{i\vec{p}\cdot\vec{x}} G(\vec{x})$$

With "Diff. op" $= \nabla^2 + |\vec{k}|^2$, use

$$G(\vec{x}-\vec{x}') = \int \frac{d^3p}{(2\pi)^3} e^{-i\vec{p}\cdot(\vec{x}-\vec{x}')} \tilde{G}(\vec{p})$$

$$\left(-|\vec{p}|^2+|\vec{k}|^2\right)\tilde{G}(\vec{p}) = 1$$

$$\int \frac{d^3p}{(2\pi)^3} \times \cdots$$

$$\tilde{G}(\vec{p}) = \frac{1}{-|\vec{p}|^2+|\vec{k}|^2}$$

$$G(\vec{x}-\vec{x}') = \int \frac{d^3p}{(2\pi)^3} \frac{e^{-i\vec{p}\cdot(\vec{x}-\vec{x}')}}{-|\vec{p}|^2+|\vec{k}|^2}$$

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