.... "paradoxes" of QM :

In Class Mech : q & p measurable

with infinite precision

QM : Either p or q and then

"uncertainty" ??

....recall Aristotelian vs. Galilean methods

... apply to Mechanics

In Class Mech : "instantaneous" velocity

$$V = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

... never verified as per Galilean ethos !!!

Path Integral formulation (contd.)

Feynman's postulate

$$\langle x_f t_f | x_i t_i \rangle = \sum_{\text{paths } x(t)} e^{iS[x(t)]/\hbar}$$

$$S \to S[x(t); x_i t_i, x_f t_f]$$

$$= \int_{x_i t_i}^{x_f t_f} dt \, L(x, \dot{x}, t)$$

Note L. H. S. contains a transition amplitude between two position eigenstates.

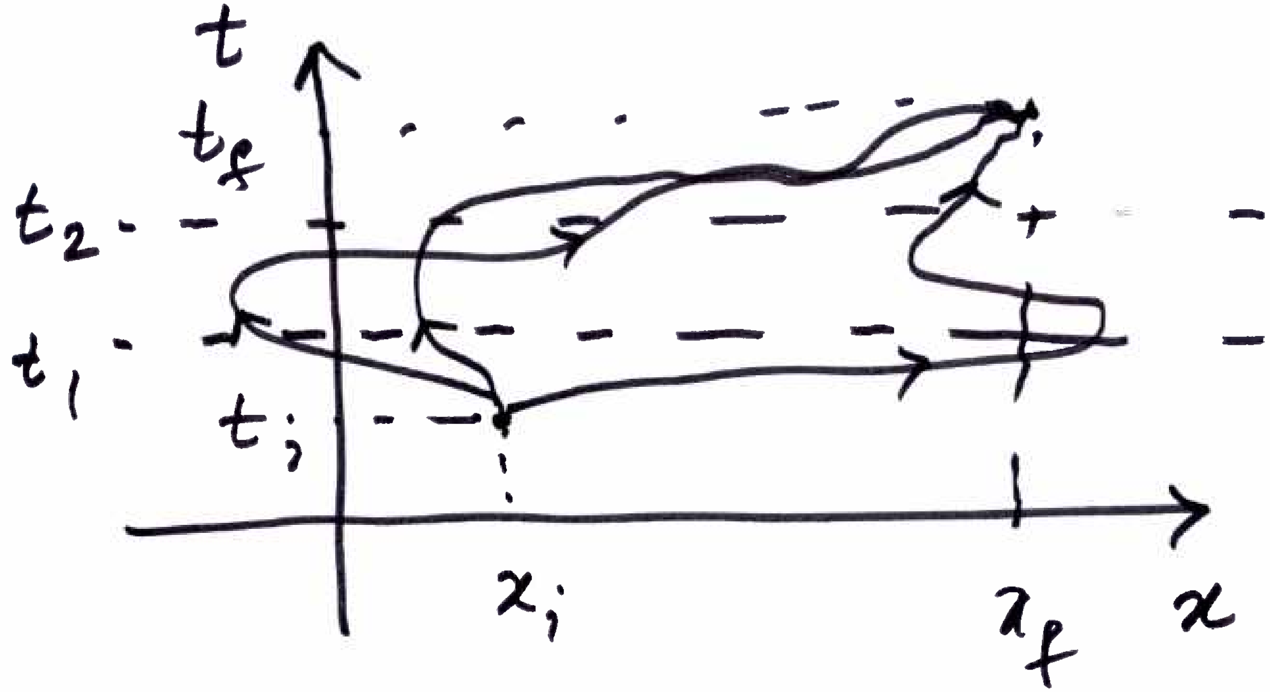
- Gives directly a transition $t_i \to t_f$

  .... can take differential limit

  $t_f \to t_i$ but this is not required

The trans. ampl. is calculated as an inner product.

Graphical interpretation:



Arbitrary x values can be assumed at any time

But t must only be increasing.

Computation in terms of "time slicing"

Insert many $(\to \infty)$ intermediate time values $t_i < t_1 \leq t_2 < t_3 \ldots < t_f$

Thus if we can calculate

$$\langle x_f, t+\Delta t \mid x_i\, t \rangle \qquad \Delta t \to 0$$

then we can combine there infinitesimal amplitudes to get final answer

Equivalence to canonical formulation

Dirac's motivation:

- Time development in Class. Mech. can also be thought of as a canonical transf. of phase space variables.

$$\dot{p} = -\frac{\partial H}{\partial q} \qquad \dot{q} = +\frac{\partial H}{\partial p}$$

- The generating function of time transl. operation is Hamilton's Princip~~le~~al Function

$$S[x(t), x_i t_i\, x_f t_f] = \int_{x_i t_i}^{x_f t_f} \{ \cancel{p\dot{q}}\, p\dot{x} - H(p,x,t)\} dt$$

where the t integral is on the classical trajectory.

- We have the rule : For a class. transf. generated by dyn. var. A, there exists a unitary transf. in QM which reproduces same transf. on the Hilbert space

$$U = e^{iA}$$

Thus Dirac concluded

$$\langle x_f\, t+\Delta t | x_i\, t \rangle \sim \exp\left\{ i\, \frac{\text{Hamilton's Ppl. func.}}{\hbar} \right\}$$