Course summary / revision

1. Path Integral

[Not for exam]

Technique:

Gaussian integrals for

$$\int \prod_i dx_i \, e^{\frac{i}{\hbar}\left(\sum_{i,j} x_i A_{ij} x_j + \sum_j B_j x_j\right)}$$

simple example: $\underbrace{Ax^2 + Bx}$ $\xrightarrow{\text{vector}}$ $x^T A x + B^T x$

Complete squares ...

$$\sim e^{B^2} \times \frac{1}{\text{Det } A}$$

Check for simplest case

$${}_D\langle x_f t_f | x_i t_i \rangle_D = \int \mathcal{D}p\,\mathcal{D}x\; e^{iS[x,p]}$$

$$\text{i.e. } \sum_{\substack{\text{paths}\\ \text{joining}\\ (x_f t_f) \text{ to } (x_i t_i)}} e^{i(\text{Action on the path})}$$

2. Peturbation Theory:

$$H_0 | n^{(0)} \rangle = E_n^{(0)} | n^{(0)} \rangle$$

Need to solve $(H_0 + \lambda V)|n\rangle = E_n^{(\lambda)} |n\rangle$

λ is for "book keeping" $\rightarrow$ matching terms of same powers of λ.

λ set to 1 after matching

In a given problem instead of λ there is usually a small physical parameter

Useful formulae:

$$\Delta E_n^{(1)} = \langle n^{(0)} | V | n^{(0)} \rangle$$

$$| n^{(1)} \rangle = \sum_{k \neq n} \frac{V_{kn} | k^{(0)} \rangle}{E_n^{(0)} - E_k^{(0)}}$$

$$\Delta E_n^{(2)} = \sum_{k \neq n} \frac{|V_{kn}|^2}{E_n^{(0)} - E_k^{(0)}}$$

For most problems $H_0 \longrightarrow$ H.O. in 1, or 2 or 3 dim

$\hookrightarrow$ Particle in 1d or 2 or 3d box and located 0 to L or −L to L

Ang. mom. problems. $\leftarrow$

J_+ J_- $|jm\rangle$

2 a) Degenerate case (need formulae only)

$\Delta E_n^{(1)} \rightarrow$ Diagonalisation of V matrix calculated in degenerate subspace of E_n

(Degeneracy may be lifted only partially)

Correction to vectors $|n\rangle$ not included in course

$$\Delta E_n^{(2)} = \sum_{k \notin D} \frac{|V_{kn}|^2}{E_n^{(0)} - E_k^{(0)}}$$

(Eq. 5.2.15 in Sakurai)

General note: Use parity or $\vec{x} \rightarrow -\vec{x}$ symmetry as general arguments

3 Variational method

→ Upper bound on g.s. energy even with no parameters.

$$\bar{H} = \frac{\langle \psi_{trial} | H | \psi_{trial} \rangle}{\langle \psi_{trial} | \psi_{trial} \rangle} \geq E_0$$

→ Can also have ψ_{trial} with parameters

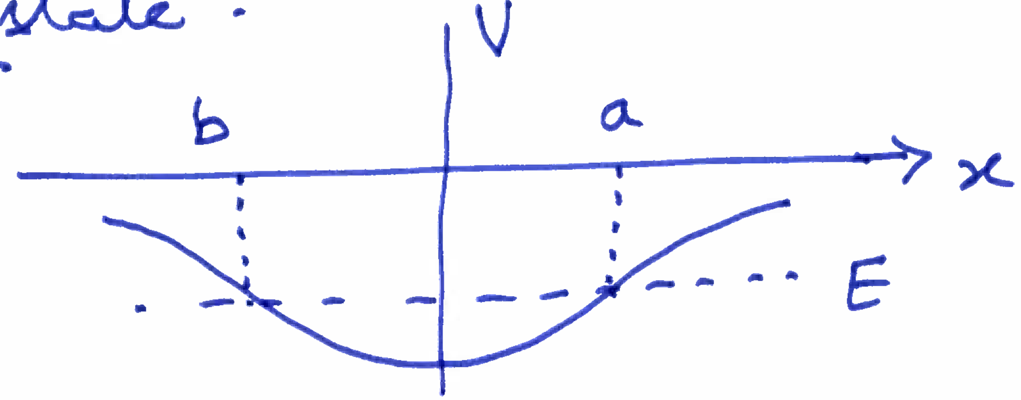
E_{best} found by extremising $\partial \bar{H}/\partial \lambda_i = 0$

ψ_{trial} to be guessed to (i) Match boundary conditions (ii) n^{th} excited state has n nodes in 1-d problems

4 **WKB** "Slowly varying" potentials $V(\vec{x})$

[Connection formulae required]

Bound state :



Classical turning points $E = V(x)$

Formula as condition for bound states

Barrier penetration : class. turning points a & b

$$T = \exp\left\{ - \int_a^b \cdots \sqrt{V - E} \cdots \right\}$$

5 Time dependent pert. th. problems

Interaction picture time dep.

coeff.s $c_i(t)$

Warm up : Solve 2-level system exactly

Then solve:

1. Harmonic pert.
2. Constant pert. turned on at $t=0$

↳ Need to develop formalism of rate W

↳ cross-section → photoelectric effect

Note $H = \frac{1}{2m}\left|\vec{p} - \frac{e}{c}\vec{A}\right|^2 + e\phi$ for E-M interaction

Revision (contd.)

6 S-matrix & scattering

Lect. 34 8th slide

Scattered wave assumed to have form

$$\chi_\alpha^+ = u_\alpha(\vec{r}) + C\,\frac{1}{r} f(\vec{k}_\alpha, \hat{r})\, e^{i|\vec{k}_\alpha| r}$$

incoming

outgoing spherical wave

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

elastic scattering

$|\vec{k}_f| = |\vec{k}_i|$ only θ matters

In Born approx for $f(\theta)$: (V is weak pot.)

$$f(\theta) = -\frac{4\pi^2 m}{\hbar^2}\tilde{V}(|\vec{q}|, \theta)$$

$\vec{k}_f - \vec{k}_i$

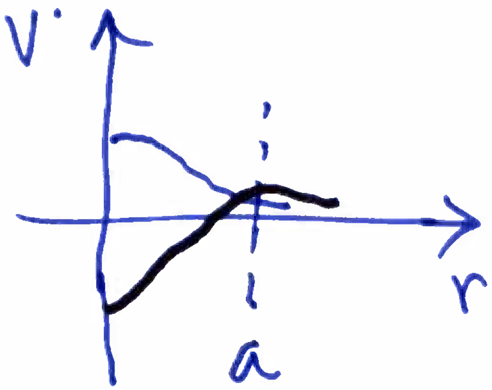
Fourier transf.

$$\tilde{V} = \int \frac{d^3x}{(2\pi)^3}\, e^{i(\vec{k}_f - \vec{k}_i)\cdot\vec{x}}\, V(\vec{x})$$

6 a) Method of phase shifts

Expansion of $f(\theta)$ in $P_\ell(\cos\theta)$ and retaining $\ell = 0$

Determine solution interior to critical distance a (outside a V is ignorable)



Match interior ψ & ψ' with exterior $A_\ell \sin(kr + \delta_\ell)$

Formula:

$$\sigma_\ell = \frac{4\pi}{k^2} |\sin \delta_\ell|^2 (2\ell+1)$$

Let $\beta_\ell = \left(\frac{r R'_{k\ell}}{R_{k\ell}}\right)_{r=a}$

then δ_ℓ determined by β_ℓ

Optical theorem: $\sigma_{tot} = \frac{4\pi}{k} \mathrm{Im} f(\theta = 0)$ || Consequence of unitarity of S-matrix