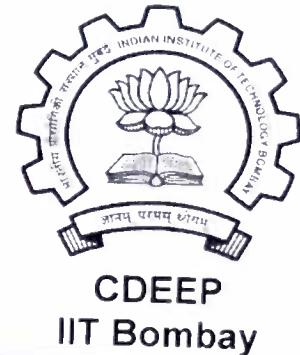


## .... on Path Integral formulation

- No obvious advantage in ~~to~~ 1-particle QM
- Suited for transition amplitudes  
... but can be adapted for a few static properties
- In QFT, useful tool for deriving formulae  
... not so much specific numericam answers
- What can be solved?
  - (i) kernel for H.O.
  - (ii) Double slit experiment?



# Bound state perturbation theory (contd.) 4

$$(\Delta E_n^{(\lambda)} - V) |\Delta n\rangle = 0$$

Thus  $\Delta E_n^{(1)} = \langle n^{(0)} | V | n^{(0)} \rangle$  to order  $\lambda$

$$|\Delta n\rangle_n = K_n \underbrace{V}_{\text{circled}} |\Delta n\rangle \rightarrow (V - \Delta E_n)$$

where

$$K_n (E_n^{(0)} - H_0)^{-1} \equiv 1 - |\Delta n\rangle \langle \Delta n|$$

$$\Delta E_n^{(\lambda)} = \langle n^{(0)} | V | n \rangle \quad || \quad (H_0 - E_n^{(0)}) |\Delta n\rangle_\lambda \\ |\Delta n\rangle_\lambda = c_0 |\Delta n\rangle + K_n (-V + \Delta E_n) |n\rangle \quad || \quad = (\Delta E_n^{(\lambda)} - V) |n^{(0)}\rangle$$



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$$(H_0 + \lambda V)(|n^{(0)}\rangle + \lambda |\Delta n\rangle_\lambda) \\ = (E_n^{(0)} + \lambda \Delta E_n^{(\lambda)}) (|n^{(0)}\rangle + \lambda |\Delta n\rangle_\lambda)$$

$$\Delta E_n^{(\lambda)} = \langle n^{(0)} | V | n \rangle$$

$$|\Delta n\rangle_\lambda = \kappa_n (\lambda V - \Delta E_n^{(\lambda)}) |n\rangle$$

Thus in first iteration,

$$\Delta E_n^{(\lambda)} = \langle n^{(0)} | V | n^{(0)} \rangle$$

$$\text{and } |\Delta n\rangle_\lambda = \kappa_n (\lambda V - \Delta E_n^{(\lambda)}) |n^{(0)}\rangle$$



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Earlier we set up

$$E_n^{(\lambda)} = E_n^{(0)} + \lambda \Delta E_n^{(1)}$$

Now } expand } =  $E_n^{(0)} + \lambda \Delta E_n^{(1)} + \lambda^2 \Delta E_n^{(2)} \dots \dots$

Similarly

$$|n\rangle_\lambda = |n^{(0)}\rangle + \lambda |\Delta n\rangle_\lambda$$

Now ~~expand~~ expand

$$\lambda |\Delta n\rangle_\lambda = \lambda |n^{(1)}\rangle + \lambda^2 |n^{(2)}\rangle + \dots$$

Then we can see that at a given order,

$$\Delta E_n^{(N)} = \langle n^{(0)} | V | n^{(N-1)} \rangle$$



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$$|n^{(1)}\rangle = \cancel{K_n V + n^{(0)}} K_n (\Delta E_n - V) |n^{(0)}\rangle$$

... Then we can go to next order for  $\Delta E$

$$\Delta E_n^{(2)} = \langle n^{(0)} | V | n^{(1)} \rangle$$

$$= \langle n^{(0)} | V K_n (\Delta E_n - V) | n^{(0)} \rangle$$

Thus we can derive

$$\boxed{\Delta E_n = \lambda \langle n^{(0)} | V | n^{(0)} \rangle + \lambda^2 \sum_{k \neq n} \frac{|V_{nk}|^2}{E_n^{(0)} - E_k^{(0)}}}$$

Detail:

$$|n^{(0)}\rangle \langle n^{(0)}|$$

$$\langle n^{(0)} | V \frac{1-P}{E_n^{(0)} - H_0} (\lambda V - \underline{\Delta E_n}) | n^{(0)} \rangle$$

$\uparrow$

$$\sum_k |k^{(0)}\rangle \langle k^{(0)}|$$

$$\langle n^{(0)} | V | k^{(0)} \rangle \langle k^{(0)} | V | n^{(0)} \rangle$$

$$\Delta E_n \langle k^{(0)} | n^{(0)} \rangle \rightarrow 0 \quad k \neq n$$



Shift in vector to first order is

$$|\Delta n^{(1)}\rangle = \sum_{k \neq n} \frac{\langle k|V|n^{(0)}\rangle}{E_n^{(0)} - E_k^{(0)}} |k^{(0)}\rangle$$



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