From General relativity to Planck scale Cosmology : the journey and its dilemmas

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1 Overview

- Principle of Equivalence -- conservation of energy density
- "No pre-geometry" -- boundary of a boundary
- Inertial frames to general coordinates -- a new principle or convenience? (General Covariance and general covariance)
- The dreaded Λ -> vacuum energy of Particle Physics
- Classical singularities -- their inevitability
- Observed Universe and its unreasonableness(es)

2 The great dichotomy

Newton's schema of ``Mechanics''

$$\vec{F} = m\vec{a}$$

but

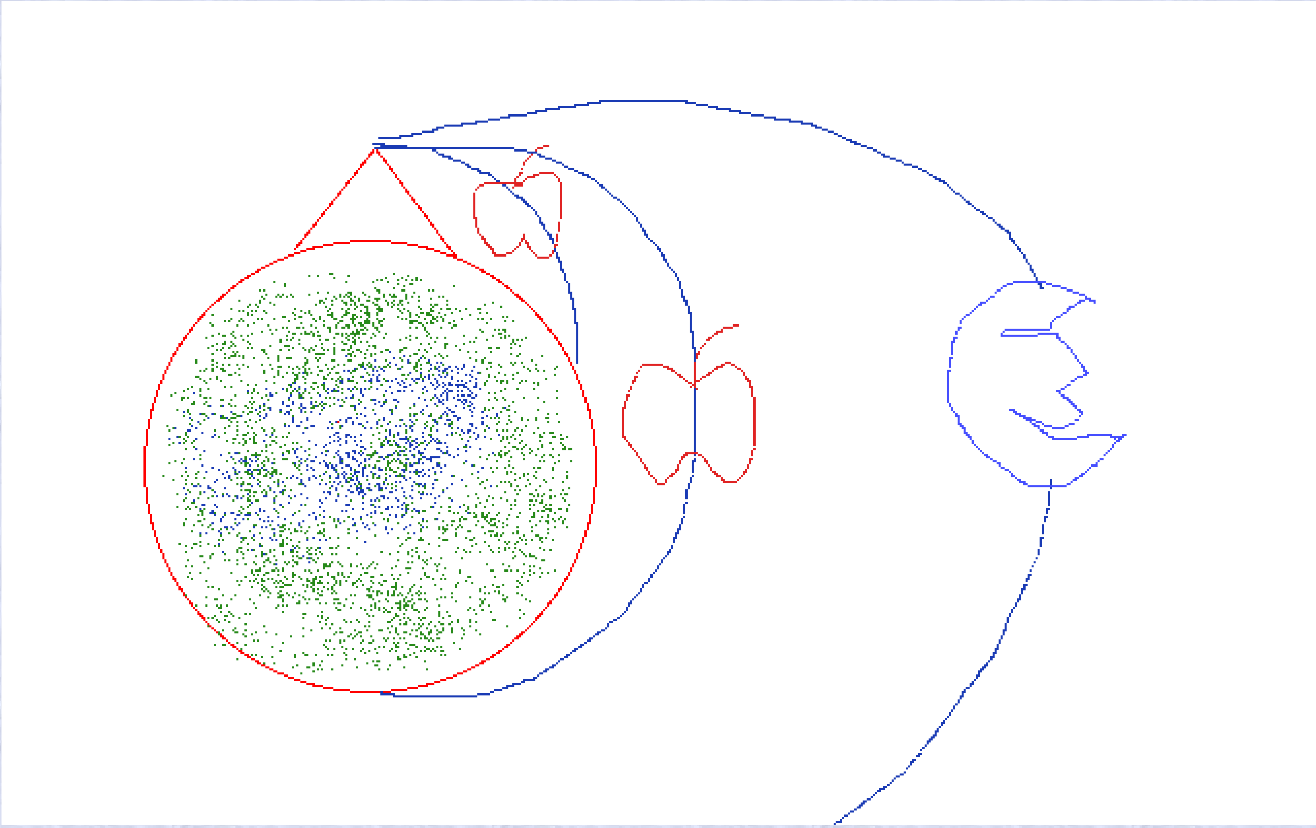
$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$$

$$\text{or......} = -kx\,\hat{x}$$

$$\text{or......} = \frac{Gm_1 m_2}{r^2}\hat{r}$$

The scheme of ``Mechanics'' disconnected from ``Force Laws''.

2.1 Newton's Universal Gravitation



→ ``Unification'' of the motions of ``terrestrial'' bodies and ``heavenly'' bodies.

→ Yet the force laws on the Earth allowed many different kinds of motions.

2.2 Newton's Gravitation ``more universal'' than expected

- In kq_1q_2/r^2, q_1, q_2 are charges specific to that force
- Likewise m_1, m_2 must be the ``charges'' of Gravitation
- But m_1 is also the inertia responding to any force $\vec{F}$.
- The ``inertia'' cancles out against the ``charge'' of the test particle

$$\cancel{m_1}\vec{a} = \frac{\cancel{m_1} m_2}{r^2}\hat{r}$$

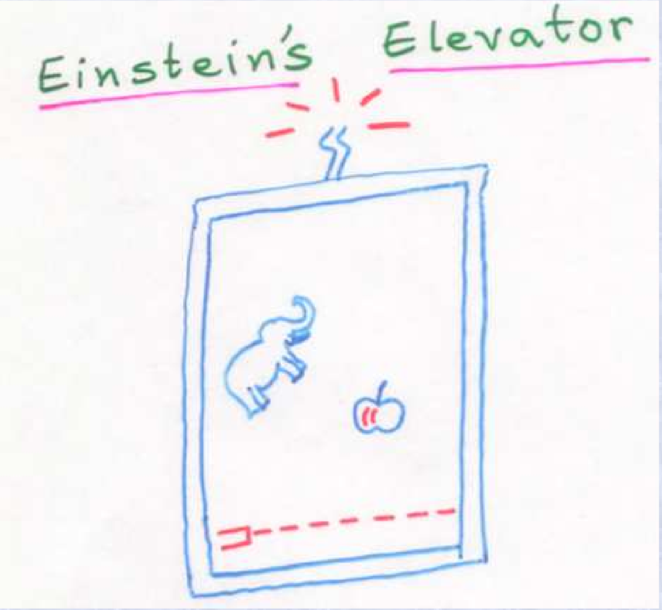
- But this happens only for Gravity, no other force.

2.3 The Equivalence Principle

Einstein's elevator

In sufficiently small regions of space-time there are choices of frames of reference in which the effects of Gravity disappear.

``Freely Falling Frames of reference'' FFFR



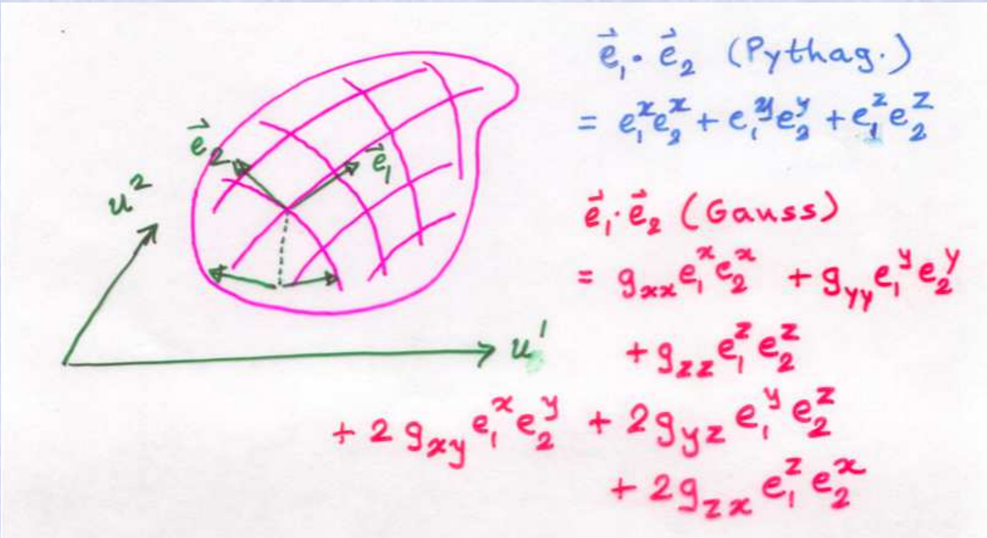
The Strong Principle of Equivalence : All the effects of gravity ... including its own dynamics

Consequence : Energy density of Gravitational field is a frame dependent concept

3 No prior geometry

All test particles fall the same way with same initial velocity --

``Curved space-time'' --> Differential Geometry



Minkowski metric :

$$\Delta s^2 = c^2(\Delta t)^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

to be replaced by the most general quadratic form

$$\Delta s^2 = g_{00}(x)\Delta t^2 + 2g_{01}(x)\Delta t\Delta x + g_{11}(x)\Delta x^2 + 2g_{12}(x)\Delta x\Delta y + \ldots$$

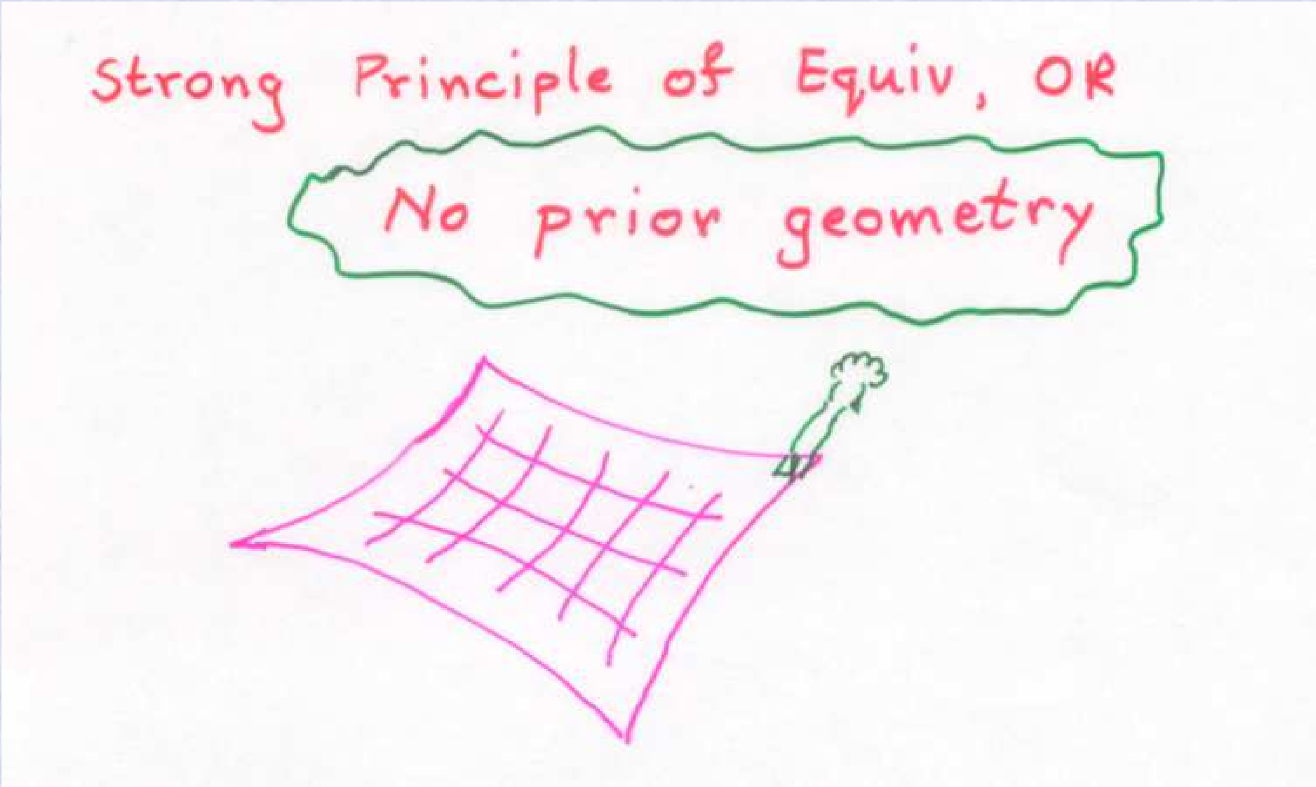
Gravity described by space-time ``metric'' coefficients.

Question :

Is the Minkowski metric a special case of metrics?

Or the $g_{\mu\nu}(x)$ coefficients are ``over and above'' the background of the Minkowski metric?

Strong Principle of Equivalence --> All of Physics, including the dynamics of Gravity itself obeys the Equivalence Principle.



If this dynamics of Gravity implies a space-time everywhere curved, no Minkowski ``background'' can be meaningfully identified!!

Einstien was inexorably led to this conclusion by staying true to the principle he had inunciated.

4 The great Einsteinian dilemma

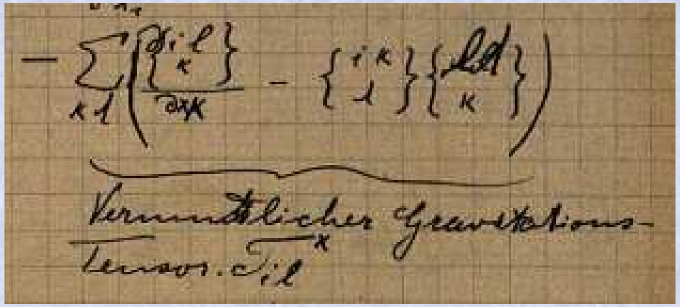
``The sought after generalisation will surely be of the form

$$\Gamma_{\mu\nu} = \kappa T_{\mu\nu}$$

where κ is a constant and $\Gamma_{\mu\nu}$ is a contravariant tensor of second rank that arises out of the fundamental tensor $g_{\mu\nu}$ through differential operations ... ...it proved impossible to find a differential expression for $\Gamma_{\mu\nu}$ that is a generalisation of [Poisson's] $\nabla^2\varphi$, and that is a tensor with respect to arbitrary transformations ... ... It seems most natural to demand that the system be covariant against arbitrary transformations. That stands in conflict with the result that the gravitational field does not possess this property." [A. Einstein and M. Grossmann, 1913]

4.1 The Einstein tensor

Possibly the $\Gamma_{\mu\nu}$ from Einstein's Zurich notebook of 1912-13



Conservation of total energy follows due to vanishing divergence,

$$\frac{\partial}{\partial x^{\mu}} T^{\mu\nu} = 0$$

--> A combinaton $\Gamma_{\mu\nu}$ of second derivatives of $g_{\mu\nu}$ with vanishing divergence is not covariant

--> A potential covariant candidate $\Gamma_{\mu\nu} \sim R_{\mu\nu}$ the Ricci tensor does not have vanishing divergence

Solution : Einstein tensor

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

which is

(i) covariant and

(ii) obeys the vanishing of its covariant divergence.

Correspondingly for the energy-momentum tensor $T_{\mu\nu}$ we get covariantly vanishing divergence and no conserved current

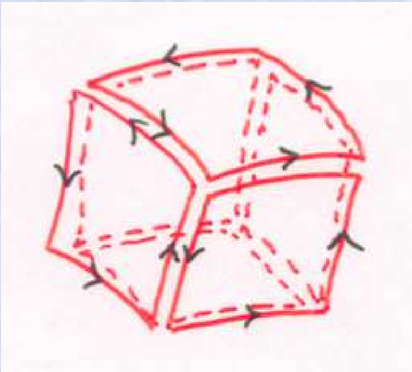
$$\frac{\partial}{\partial x^{\mu}} T^{\mu\nu} - \Gamma^{\nu}_{\mu\rho} T^{\mu\rho} - \Gamma^{\mu}_{\mu\rho} T^{\nu\rho} = 0$$

As we noted earlier, this is a consequence of the Equivalence Principle. In a FFFR, the effects of gravity have to go away so local energy density is not physically meaningful.

4.2 Boundary of a boundary ...Bianchi identity

While we lost something very precious in physics, (Energy!!!), we draw solace in preserving deep truths of geometry.

A mathematical identity concerning the Riemann tensor from which the Einstein tensor is made up, is a statement ``Boundary of a boundary is zero".



- That the energy-momentum density obeys the vanishing of only the covariant divergence is because it exchanges energy with the gravitational field.
- But the energy density of gravitational field has to vanish in a FFFR. So it is not a covariantly meaningful quantity.
- In turn, the curvature side obeys these identities because of the meaning assocaited with the Riemann tensor in which the round trip of a vector around a closed curve is captured.
- And repeating the operation we can make the vactor traverse closed curves which bound surfaces that bound a closed volume.

આઈન્સ્ટાઈનની દુવિધા

- ભૂમીતિ વિરુદ્ધ ભૌતિકી
- ભૌતિકીના સિદ્ધાન્તો મુજબ ઊર્જાનું સંરક્ષણ થવું જોઈએ
- ઉપરોક્ત સમીકરણોમાં ઊર્જાનું સંરક્ષણ થતું હોય તો ભૌમિતિક અર્થઘટન અસમ્ભવ બને
- ભૂમીતિમાં અમુક પાયાના નિયમો સ્વયંસિદ્ધ છે, જેને બિયાન્કીનાં સમીકરણો કહેવાય છે
- આ સમીકરણો સંરક્ષણ સમીકરણો જેવાં દેખાય છે, પણ તેમાંથી કોઈ સંરક્ષણ નિયમ ઉદ્ભવતો નથી

ભૂમીતિ ચઢે કે ભૌતિકી?
કુદરતને શું મંજૂર છે?

5 The principle and its practice

Principle of Equivalence ≡ General Covariance

General Covariance ≡ Reparametrisation Invariance + metric

The metric coefficients $g_{\mu\nu}(x)$ are in turn to be determined by dynamical equations that respect the Principle of General Covariance.

How unique is the Principle in determining the equations determining $g_{\mu\nu}(x)$?

The Einstein Equations are not unique!!

5.1 Hierarchy of geometric concepts

→ Topology

→ Manifolds

→ Affine connection

→ Curvature and torsion

→ Metric compatible OR NOT with the affine connection

General Relativity uses quasi-Riemannian manifolds

- Should torsion be included?
- Are the determining equations at most quadratic in derivatives, like the usual dynamical systems?
  - The higher derivative terms can be presumed to be subdominanat at large scales and dominanat at smaller scales
  - Thus at planetray, terrestrial, atomic, nuclear, quark ... scales these effects woudl get stronger

- No such smaller scale are seen, down to the LHC hence ...⇒ such terms if at all are relevant at Planck scale and can be ignored for now
- They may however play a role near the Big Bang, sa is being done for Infaltionary proposals.

• How about ``lower'' derivative term?

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

This is the infamous Cosmological Constant. Its effects could possibly be seen on scales much larger the planetary scales.

- Einstein proposed that it would have an effect on the largest possible scales and keep the Universe from being ``unstable''.

- For close to a century no effects of a possible cosmological constant were to be observed.
- On the other hand the Universe was found to be expanding by Edwin Hubble in 1929.

5.2 Singularities!

- ``No prior geometry'' principle buys us more than what we ordered.
- Metric coefficients become dynamical quantities.
- Space and time of the oberver are themsleves getting modified as she plans and proposes to make any observations ....

- And if you wait too long … you may meet the unfortunate fate of being scrunched into a singularity

- Singularities inevitable to Gravity

So how do find a theory that prevents this unphysical consequence?

Or is it Physical?

5.3 To summarise the issue of the Principle vs Practice :

- We may think the power of the Principle of Equivalence to be limited, in that the presence of terms relevant to the very small scales and very large scales remain ambiguous.

- On the positive side, the principle does strongly restrict what kind of new terms that will have to be considered when such phenomena are explored.

- In any case we seem to be stuck with classical singularities.

6 The unreasonableness of the observed Universe

- Expanding Universe predicted 1922; observed 1929

- Inflationary Universe recognised 1980

- Accelerating Universe observed 1998

6.1 The three laws of Cosmology

An unstable Universe should have been anticiapated from purely attractive nature of Gravity!

So we need not be too suprised about the Universe according to Friedmann and Hubble -- also Lemaitre (JVN's talk on History of GR)

But the real unreasonableness is the lengthscale and the time scale of the Universe. Consider the Friedmann equation :

- Law I : Simplified metric for homogeneous and isotropic space-time

$$\Delta s^2 = c^2 \Delta t^2 - a^2(t) \left(\frac{\Delta r^2}{1 - k r^2} + r^2 \Delta\theta^2 + r^2 \sin^2\theta \Delta\varphi^2 \right) \qquad k = -1, 0, +1$$

- Law 2 : The Friedmann evolution equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3} G\rho$$

- Law 3 : ``Conservation'' of energy + Equation of state $p(\rho)$

$$d(\rho a^3) + p\,d(a^3) = 0$$

with $p = w\rho$ where w depends on the substance

This is like $dU + pdV = 0$ of Thermodynamics

Scale factor $a(t)$ can be found for

(i) $p=0$ nonrelativistic matter $a(t) \propto t^{2/3}$

(ii) $p=\frac{1}{3}\rho$ purely radiation $a(t) \propto t^{1/2}$

(iii) $p=-\rho$ vacuum energy $a(t) \propto t^{H_0 t}$

6.2 A problem of scales

Three terms in the Friedmann equation

$$\left(\frac{d}{dt}\right)^2, \quad \frac{1}{a^2}, \quad G\rho \sim \frac{(M_{mattter})^4 or T^4}{M_{Pl}^2}$$

What should M_{matter} or generic temperature T be?

Note $G^{-1/2} \sim M_{Pl} \sim 10^{19}$GeV!!!

If we put M_{matter} to be 100 to 200 GeV, like the Higgs particle found at the LHC, or the top quark, then we get

Energy scale of ~10^{-15}GeV; or 10^{14}fermi, or 0.1m; or … $3x10^{-10}$sec

એક નેનોસેકન્ડમાં વિશ્વનો ખેલ ખતમ!

But the age of the Universe today is 14Byr$\sim 3\times10^{17}$sec

How do we get such extremely large time scale??

By having extremely small mass scale ...

But we also need to fine tune it to this value to same extent of

$$\frac{\delta\rho}{\rho} \sim 10^{-72}$$

or so, in order to arrive at today's Universe starting from the Planck scale.

6.3 Inflationary Universe

One proposal to avoid all this fine tuning is called ``inflation'', or an era of exponential expansion.

I highlight only a few conceptual issues

- Gravity theory alone, as know classically, with known forms of matter is insufficient to explain the behaviour near the Big Bang
- WIll Quantum Gravity solve the problem?
    - If so we lose verifiability ... no signatures of that process will be found
    - But a solution invoking known physics could be testable

The more widely accepted proposal is to invoke some non-Gravitational physics to intervene -- typically parameterised in terms of a scalar field called the ``inflaton''

7 Conclusions

- Equality of gravitational and inertial masses ⇒ Principle of Equivalence
- The stronger version of the Principle provides the equations of gravity itself … but leaves the presence of a few terms like the Cosmological constant undecided.
- There is a tussle between physical principle of Energy and the geometrical principle of ``boundary of a boundary''. Geometry wins !!!
- But once geometry wins one is left with several unpleasant consequences.
    - No ``prior geometry'' forces on us singular solutions

- The Three Laws of Cosmology as a aprt of ``no prior geometry'' package.
  - Also forces on us a singular origin of the Big Bang
- Observational challenge : The extremely large time scale fo the Universe --> The need for the ``inflaton''.
- We did not even talk about Quantum Gravity and the meaning of hte state function for the universe, the meaning of time near the big Bang etc.
- Yet the Principle of Equivalence is one of the most powerful and more elegant we know. We will be reluctant to let go of it in a hurry.

Thank you!

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