Cosmology and General Relativity : a perspective

URJIT A YAJNIK, *Physics Department, IIT, Bombay*

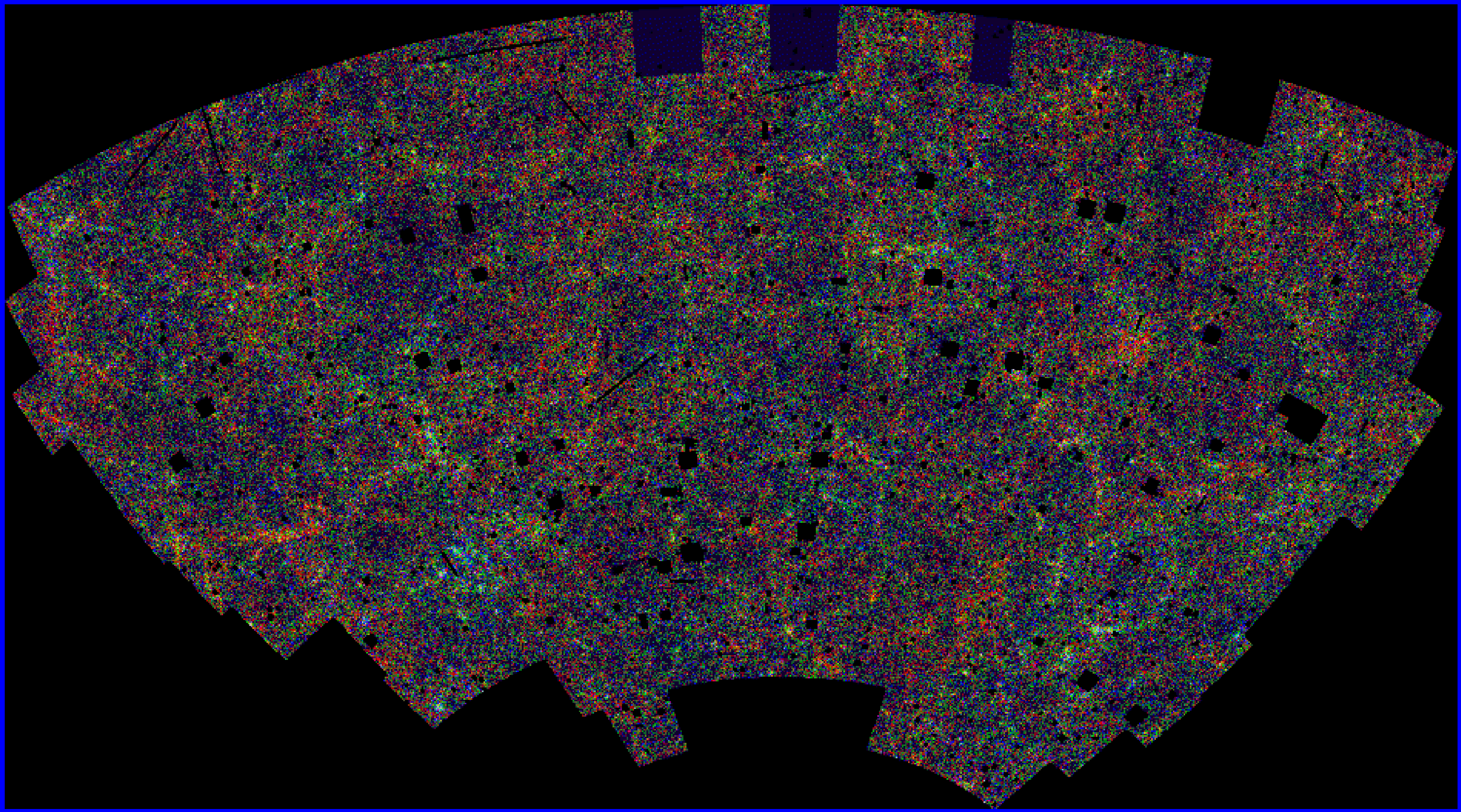
Prof. R. V. Kamath Memorial Lecture, St. Xavier's College, January 29, 2007

Outline

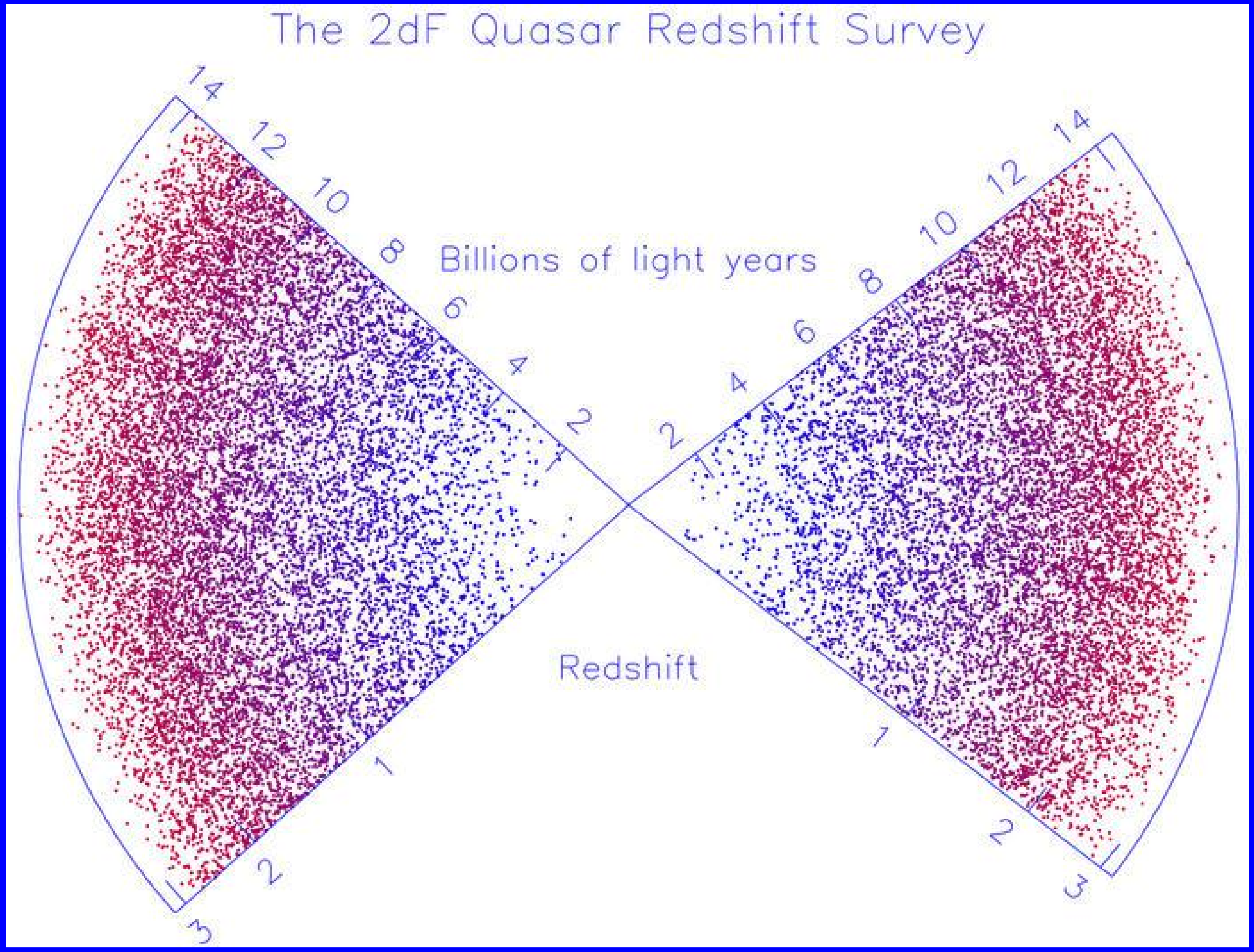
- The Three Laws of Cosmology
  - ⋆ Hubble plots and galaxy, quasar distributions
  - ⋆ The Cosmological constant
- The need for Inflationary Universe
- WMAP confirms inflation
- Action Principle for General Relativity – a page from history

The Universe observed

Distribution of galaxies (2-degree-Field survey)



Quasar distribution



The Three Laws of Cosmology

Gravity = curved space-time

Scale-factor: The Universe appears to be homogeneous and isotropic and hence is described by the following generalisation of the space-time interval

$$ds^2 = dt^2 - R(t)^2\{\frac{dr^2}{1+kr^2} + r^2 d\theta^2 + r^2 \sin^2 d\phi^2\}$$

$k = 0$ for flat Universe; $k = \pm 1$ for constant positive or negative curvature

$R(t)$ the Scale factor ... A. A. Friedmann

Equation-for-scale-factor: The dynamics of R is determined by the total energy density ρ

$$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 + \frac{k}{R^2} = \frac{8\pi}{3}G\rho$$

Note : the combination $\dot{R}(t)/R(t)$ will be denoted $H(t)$. It signifies the expansion rate of the Universe in intrinsic length units. Its present value is the Hubble Constant H_0

Equation-of-state: We need to specify the pressure-energy-density relation $p = p(\rho)$

Usually $p = w\rho$

Examples :

1. Radiation dominated Universe : $p = \frac{1}{3}\rho \Rightarrow R(t) \propto t^{1/2}$
2. Matter dominated Universe : $p = 0 \Rightarrow R(t) \propto t^{2/3}$
3. Vacuum energy (Cosmological Constant dominated) : $p = -\rho \Rightarrow R(t) \propto e^{Ht}$

On second thoughts ...

.... add a Λ (Einstein 1924) in the law for $R(t)$ to avoid expanding / contracting Universe.

$$H(t)^2 + \frac{k}{R(t)^2} - \Lambda = \frac{8\pi G}{3}\rho(t)$$

✔ This introduces a new fundamental constant of nature, of dimensions $[L^{-2}]$,the Cosmological Constant

✔ If the $\Lambda$is transferred to the right hand side, it looks like a contribution to ρ, satisfying the unusual equation of state $p = -\rho$.

- ✘ By 1929 Hubble's Law emerges and by 1936 Einstein admits, it was the "biggest blunder" of his life to have inroduced Λ term.

Book keeping of Cosmic contents

another way of writing ...

$$1 + \frac{k}{H^2 R^2} = \Omega_\Lambda + \Omega_\rho$$

- Today LHS seems to be 1
  - ⋆ So in the curvature term, $k = 0$

Current best fit to data

- Λ term seems to dominate, $\Omega_\Lambda = 0.7$
- But most of ρ is not baryons! Let $\Omega_\rho = \Omega_{DM} + \Omega_B$
  - ⋆ Baryons contribute only $\Omega_B = 0.03$
  - ⋆ $\Omega_{DM} = 0.27$ So much is the “Dark Matter”

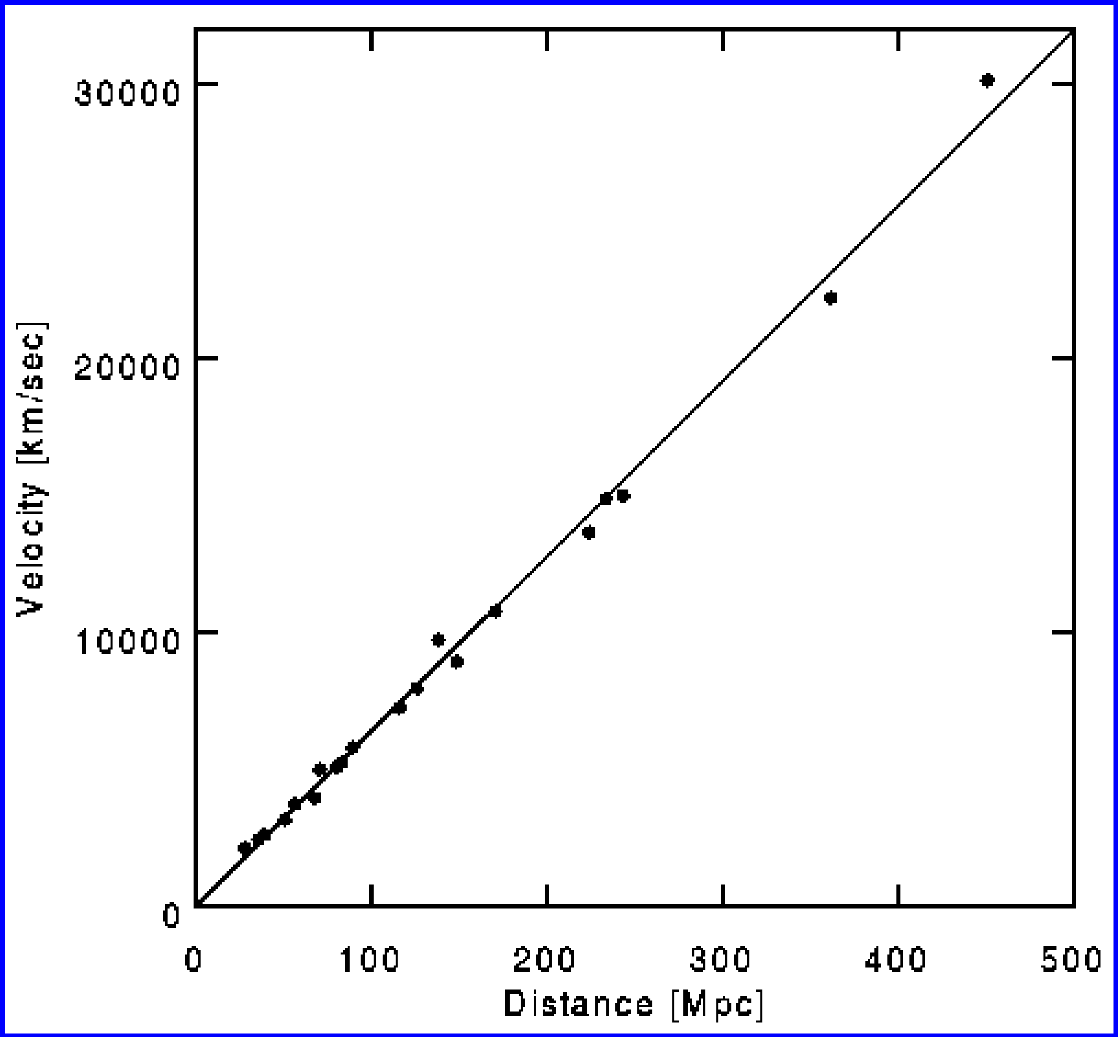
How do we know all this?

Begin in the 1920's ...

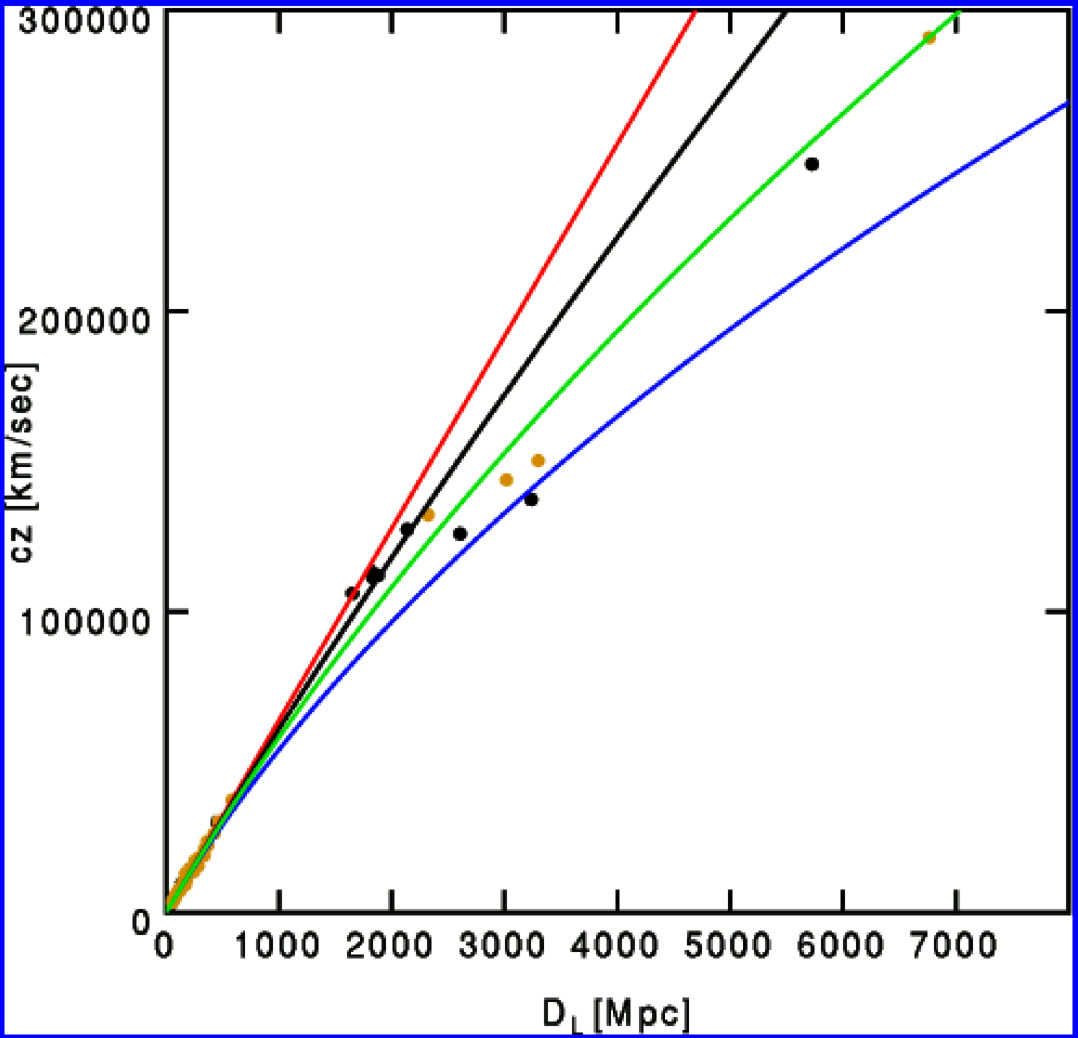
From the galactic data collected from newly deployed large telescopes, it appeared that all all but a few of the $20+$ galaxies showed a redshift rather than a blueshift.

Edwin Hubble drew a line through redshift vs. luminosity distance plot.

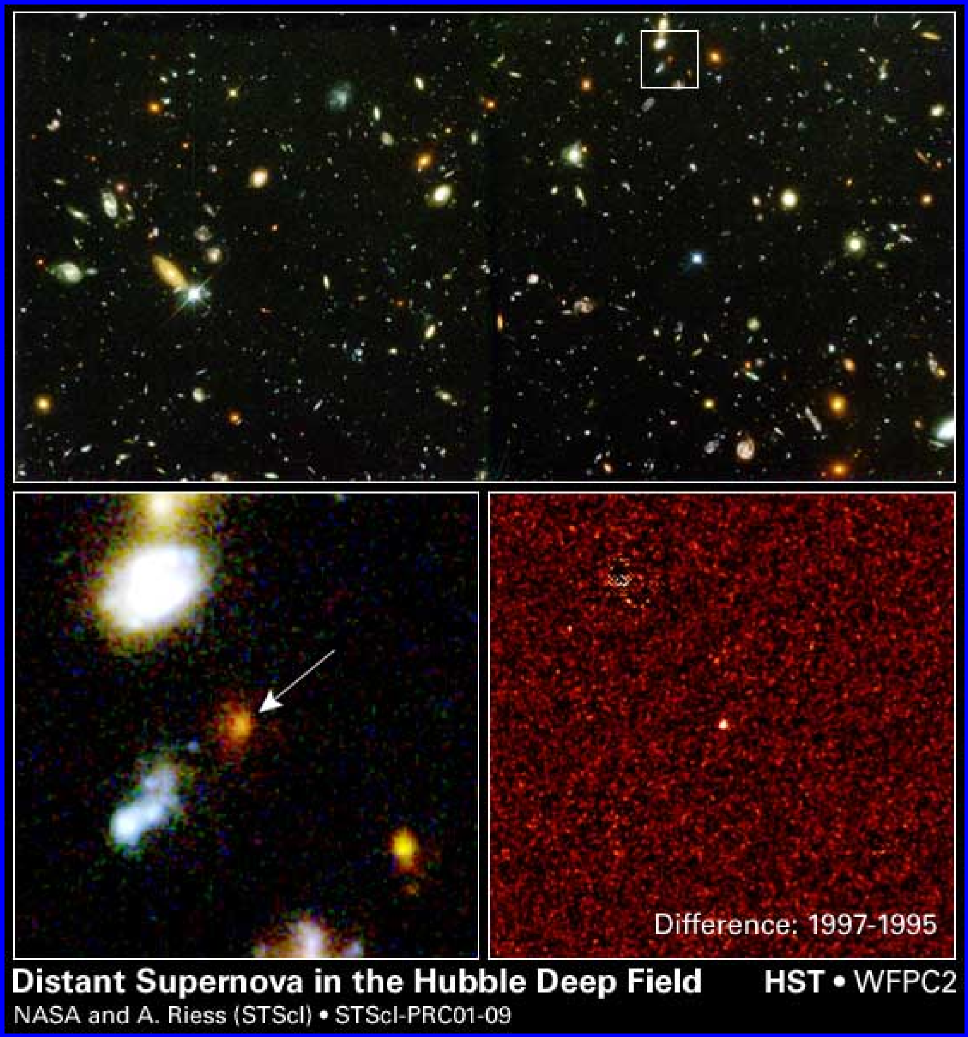
A Hubble Plot before Hubble Space Telescope



Modern Hubble plot



A blast from the remote past



Show movie

The Big Bang

The Cosmic Expansion

Extrapolated sequence backwards in time

• Ionised Hydrogen	1 eV	10^4 K
• Free neutrons and protons	1 MeV	10^{10} K
• Quark-Gluon plasma	1 GeV	10^{13} K
• Electroweak scale	100 GeV	10^{15} K
• Quantum Gravity		10^{19} GeV

Neutral H formation $\sim 10^5$years after the Big Bang

Relic radiation 10^4 K then; 3 K now

Alpher, Bethe and Gamow (1942)

Cosmography : A summary

Current parameters of the Universe :

- Expansion rate 71 ± 4 (km/s)/MegaParsec
- Size of the visible Universe 3 GigaParsec
- Age of the Universe 13.7 ± 2 GigaYears
- Age at decoupling $380 \pm 7 \times 10^3$ Year

Inflationary Universe

A problem of scales

We expect a physical system to be governed by intrinsic scales.

eg. Sizes of animals, mountains, solar system, galaxies ...

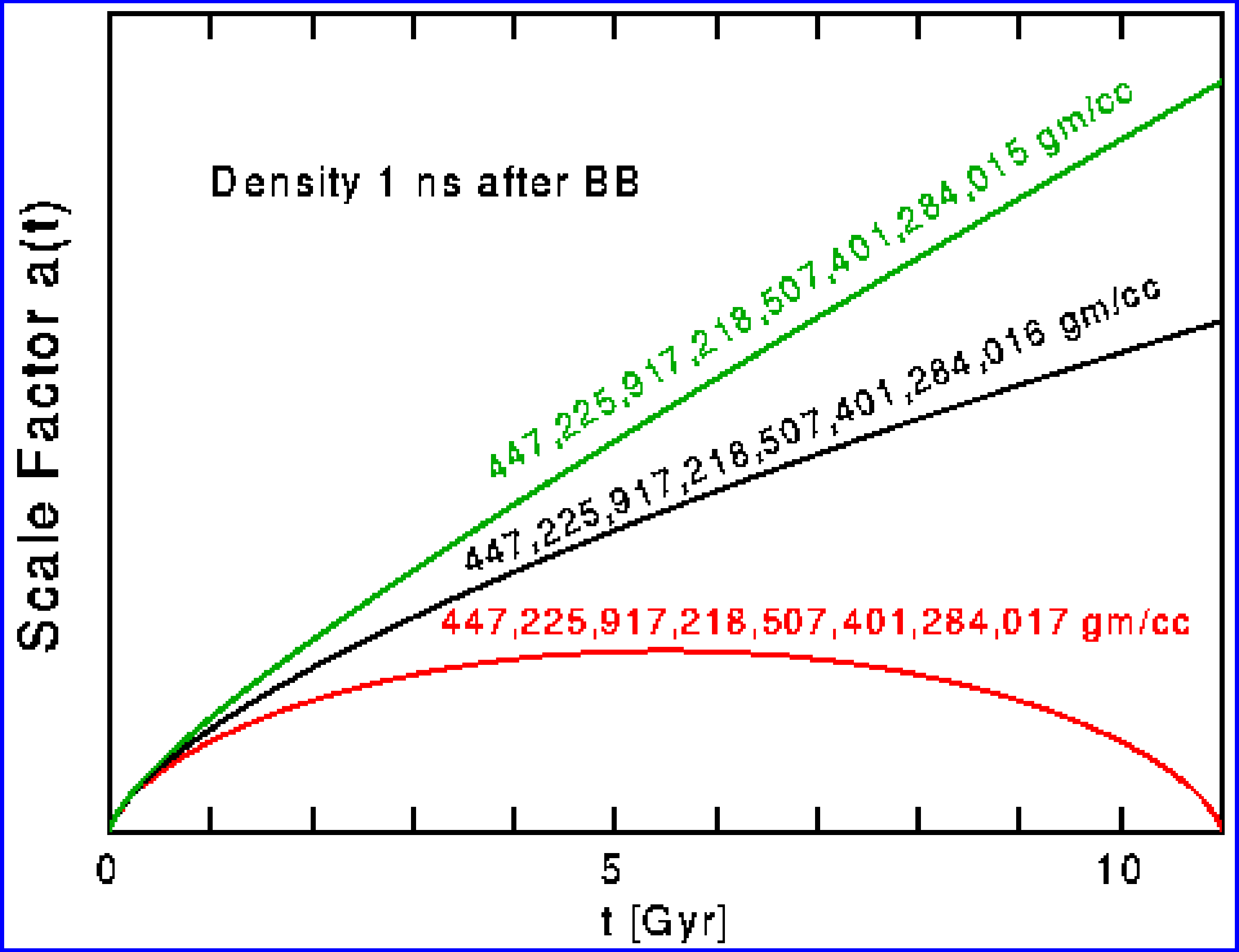
Such scales appear as (dimensionful) constants in the laws determining the state of the system

A system far too large or far too long lived compared to such intrinsic dimensions suggests ignorance of

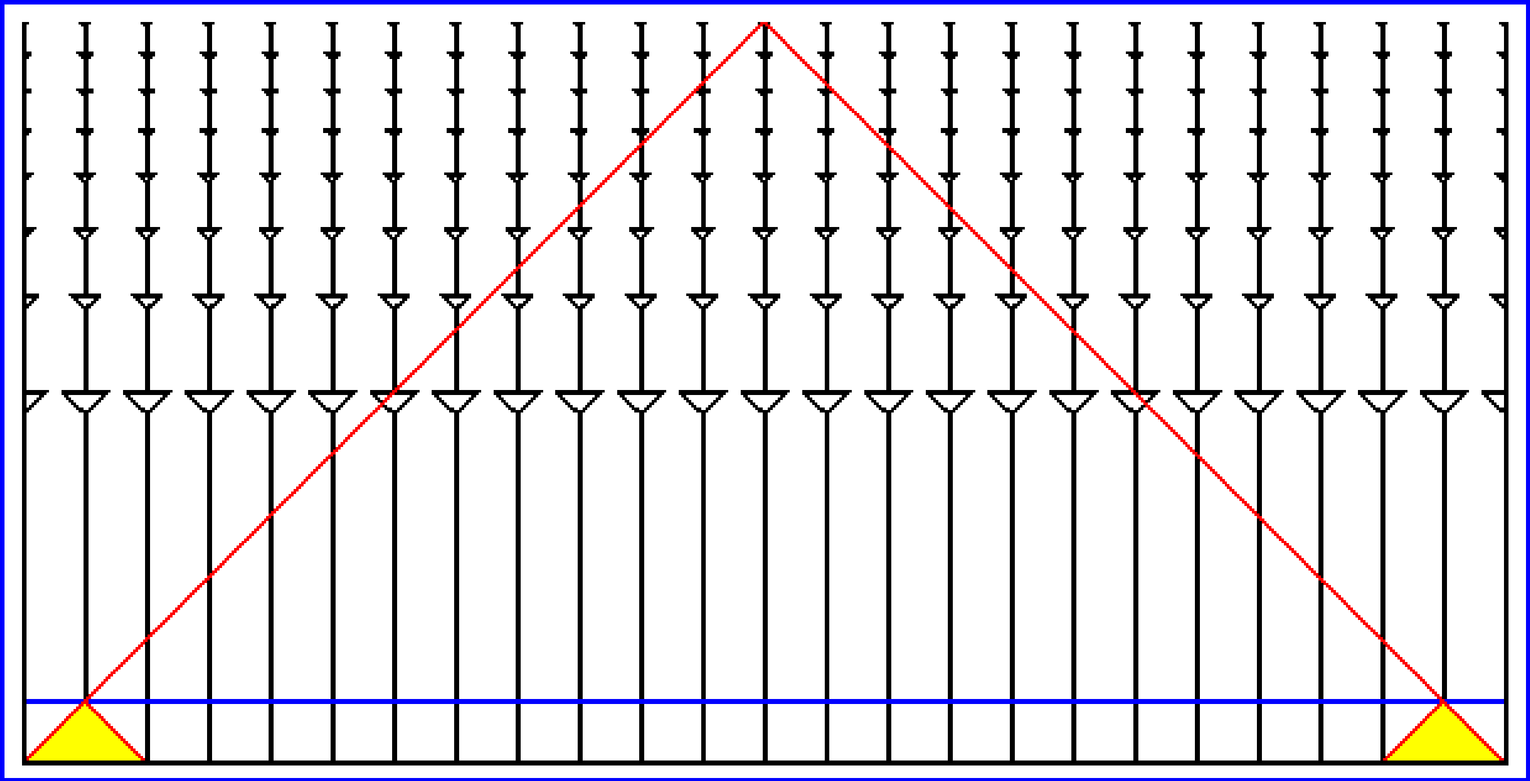
- Newer dynamics, or more importantly,
- Newer laws of nature

Oldness flatness problem

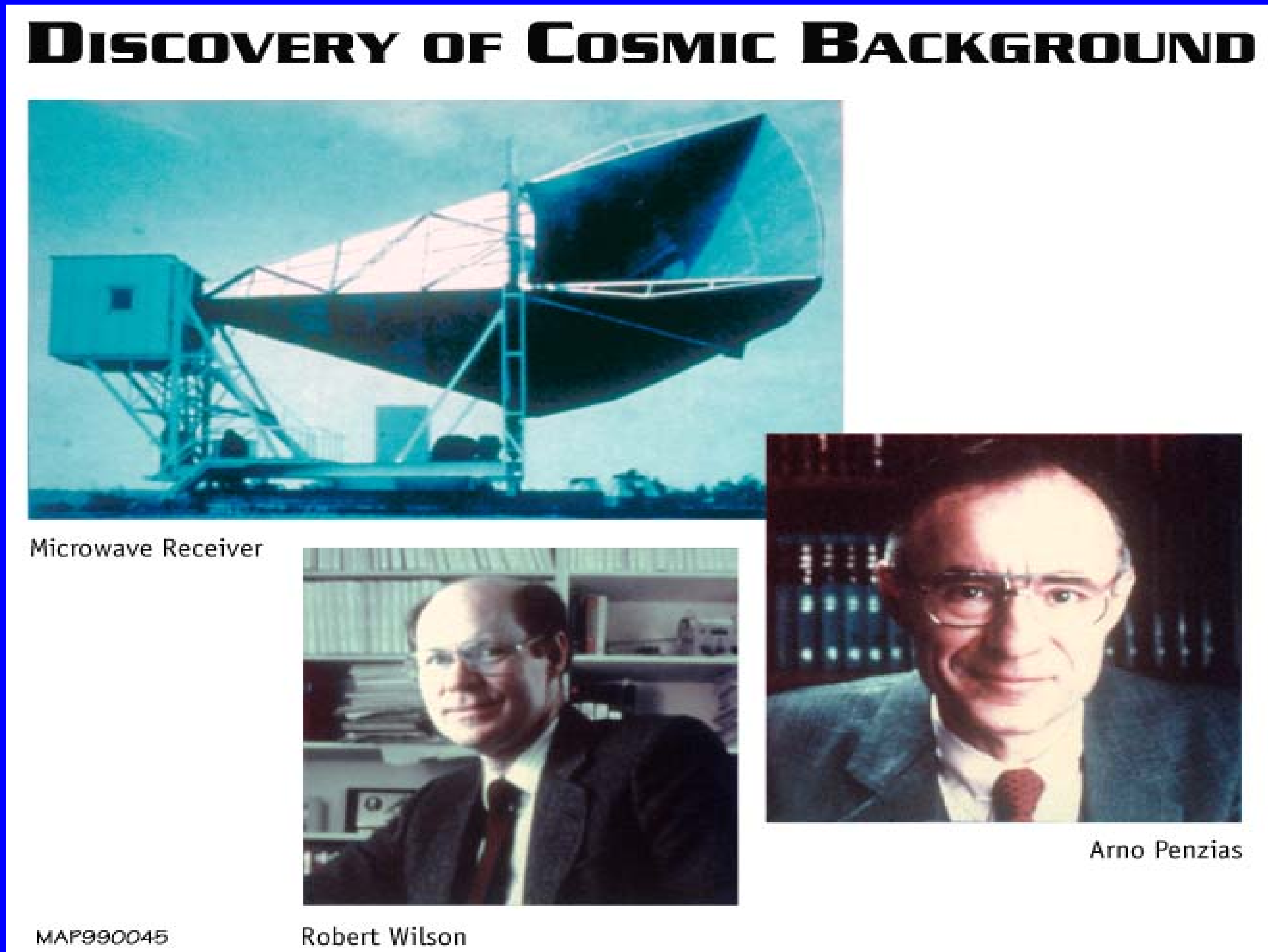
Inflation figures from Ned Wright's Online Cosmology Tutorial page


Scale Factor a(t)
Density 1 ns after BB
447,225,917,218,507,401,284,015 gm/cc
447,225,917,218,507,401,284,016 gm/cc
447,225,917,218,507,401,284,017 gm/cc
0
5
10
t [Gyr]

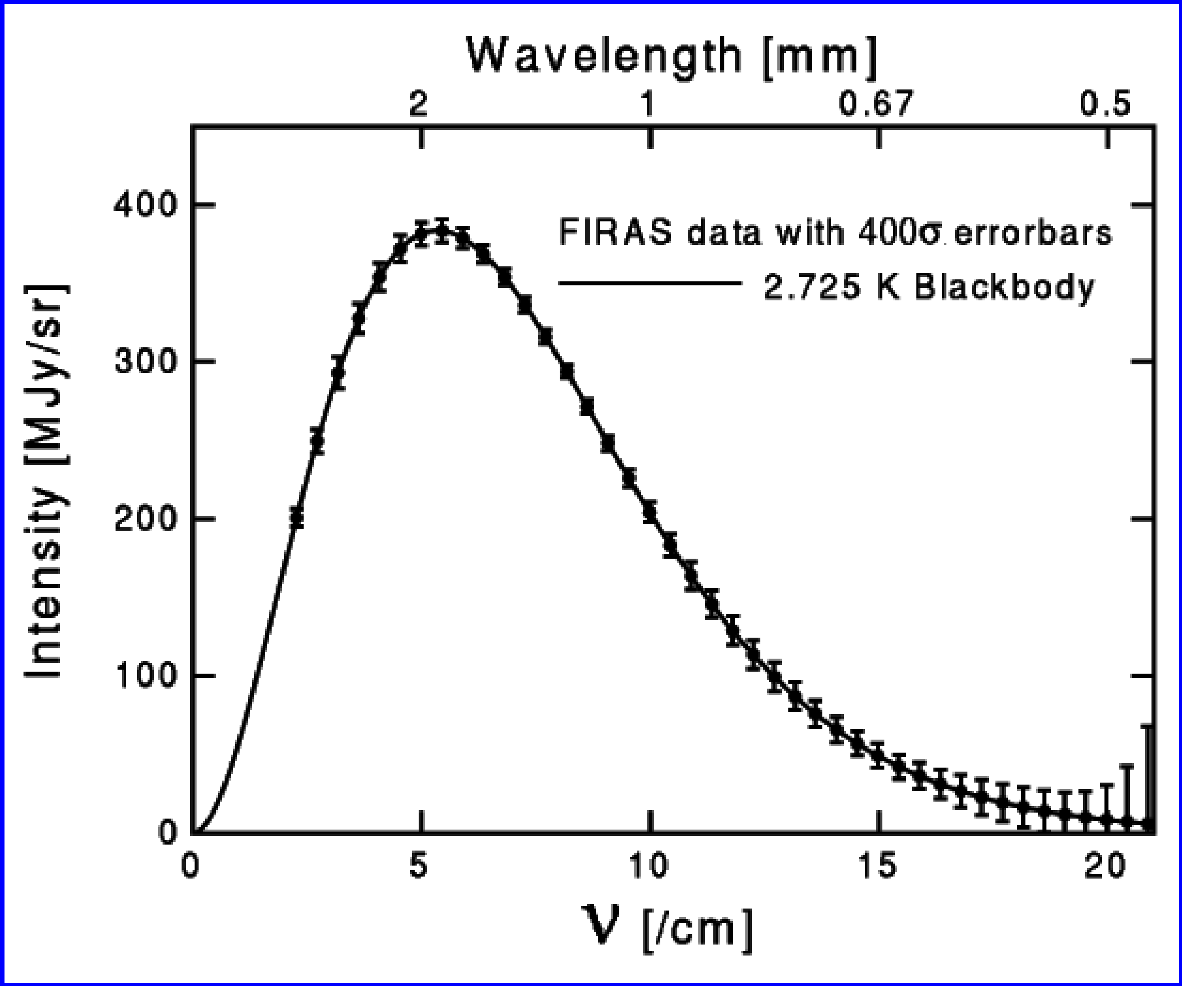
Horizon problem



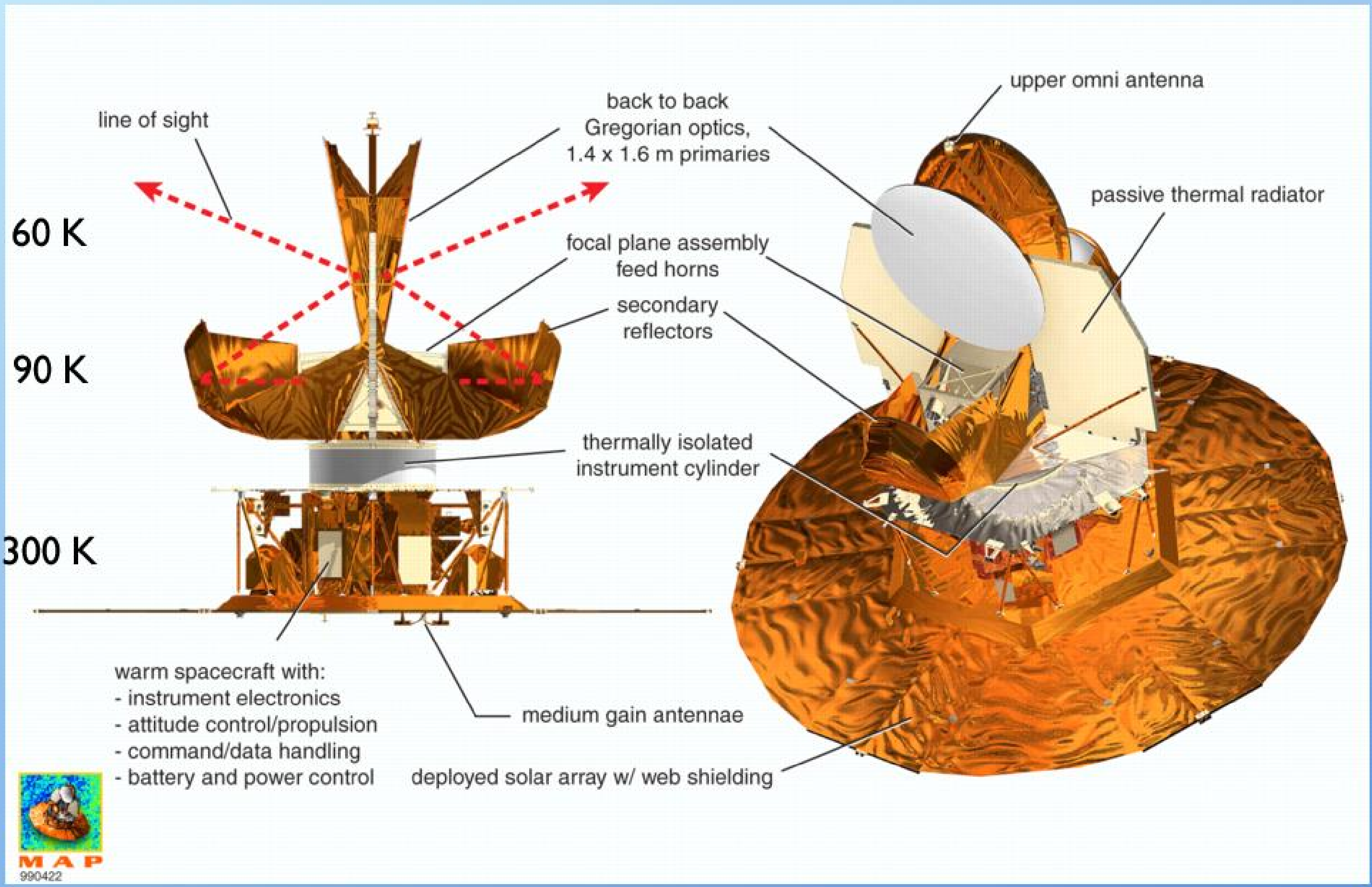
WMAP : fingerprinting the Universe


DISCOVERY OF COSMIC BACKGROUND
Microwave Receiver
Robert Wilson
Arno Penzias
MAP990045

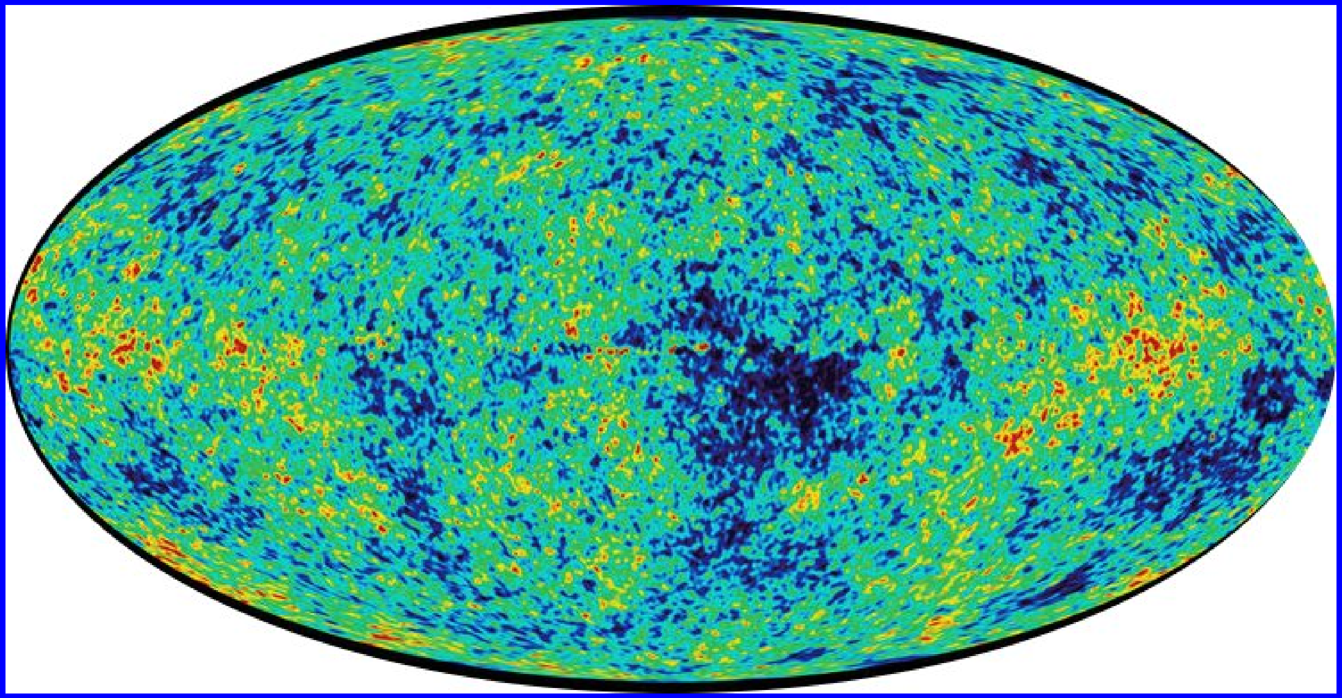
COBE data (1992)



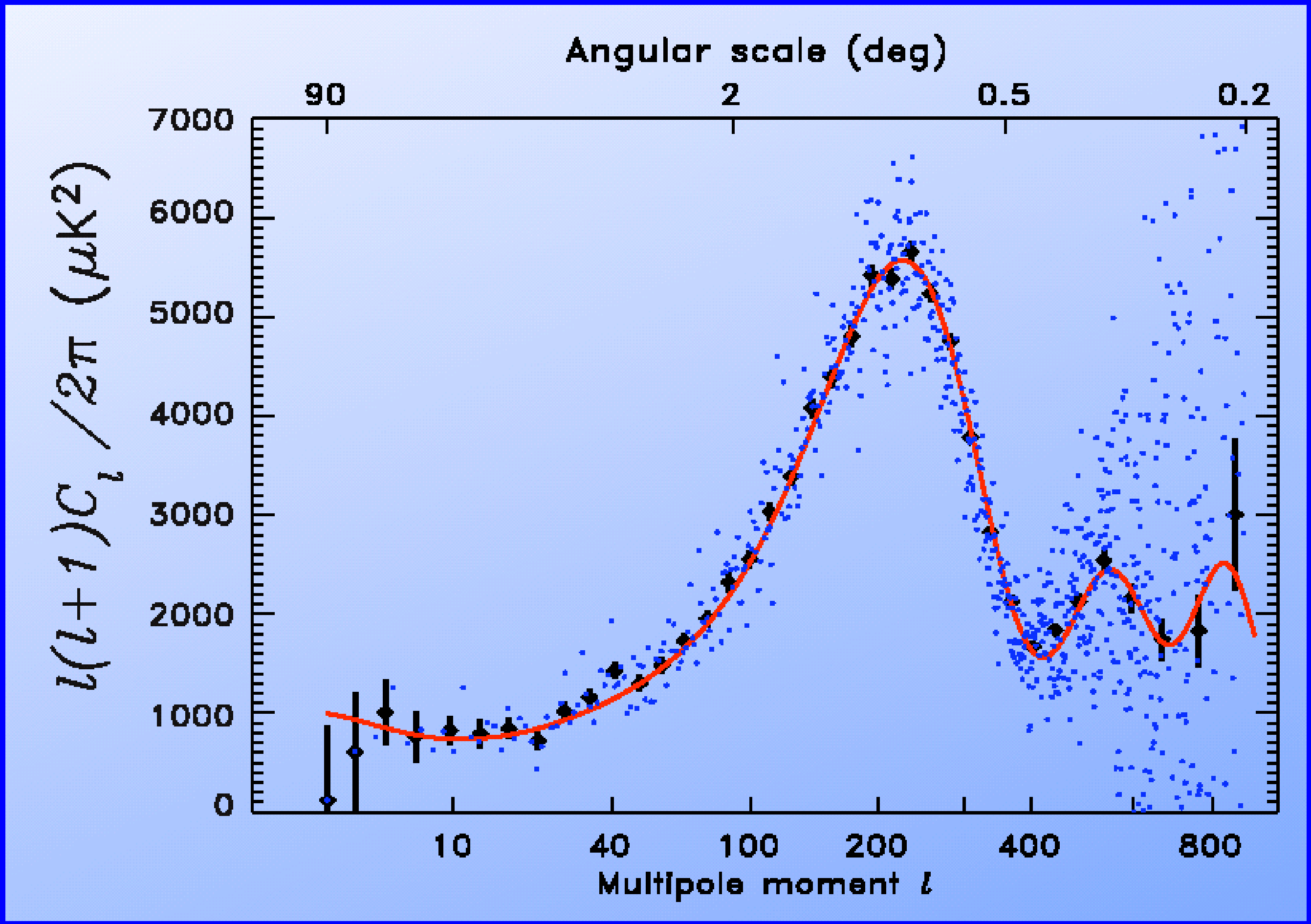
WMAP Spacecraft and Instrument



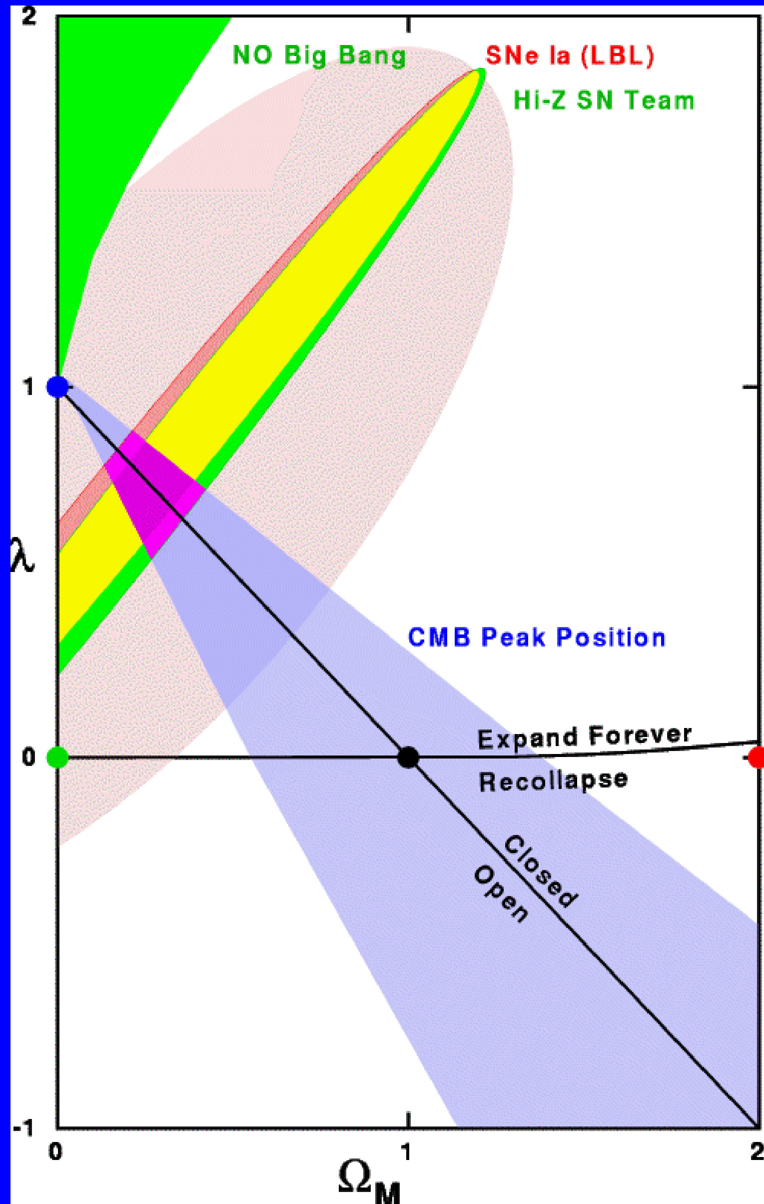
All sky microwave map of the Universe



Angular power spectrum of fluctuations



Combining supernova data and WMAP


NO Big Bang
SNe Ia (LBL)
Hi-Z SN Team
CMB Peak Position
Expand Forever
Recollapse
Closed
Open
λ
Ω_M
2
1
0
-1
0
1
2

Inflation : “before” or “after” the Big Bang?”

WMAP has fingerprinted the sky in the microwave

- We have an extremely homogeneous background
- We also have exactly the 1 in 10^5 perturbations needed to form galaxies

What gave rise to the perturbations? No interactions among particles we know about, all the way upto GUT scale can give rise to the fluctuations exactly as they are observed.

Some collective phenomenon and communication beyond the horizon is necessary

Two qualitatively different possibilities exist :

✔ GUT scale phase transition

✔ Quantum Gravity era legacy

Inflationary scenrios typically envisage a scalar field meandering around just after the Planck scale or even overlapping the Planck era, and dominating the energy density of the Universe.

After the Universe returns to being driven by usual kinds of energy – radiation or matter – we get the appearance that length scales far outside each other's particle horizon are correlated.

Inflation also predicts that mere quantum fluctuations of the primordial soup get stretched and become macroscopic, scale invariant fluctuations consistent with distribution of galaxies. No extreneous method required to generate density perturbations.

Two problems of inflationary models

- ✘ How does the inflationary epoch end?
  - ✘ Do we retain all the desirable predictions of Hot Big Bang cosmology at the end?
- ✘ The spectrum of perturbations predicted seems just right but the magnitude is too large.

“Invention” of General Relativity

Gravity = curved space-time

The Equivalence Principle $m_{grav} = m_{inertial}$

Trajectories of all test particles depend only on their initial velocites, independent of their masses.

Gravity = curved space-time

The Equivalence Principle $m_{grav} = m_{inertial}$

Trajectories of all test particles depend only on their initial velocites, independent of their masses.

General Relativity the theory of the space-time

$$\begin{aligned} \Delta s^2 &= g_{00}\Delta t^2 - 2g_{01}\Delta t \Delta x_1 + g_{11}\Delta x_1^2 + \ldots.. \\ &= \Delta x^T \mathbf{g} \Delta x \end{aligned}$$

$$= \sum_{\mu\nu} g_{\mu\nu} \Delta x^{\mu} \Delta x^{\nu}$$

The coefficients $g_{\mu\nu}$ are called metric coefficients and give the scale of length and angle measurement but in General Relativity are dependent on space and time.

$$= \sum_{\mu\nu} g_{\mu\nu} \Delta x^{\mu} \Delta x^{\nu}$$

The coefficients $g_{\mu\nu}$ are called metric coefficients and give the scale of length and angle measurement but in General Relativity are dependent on space and time.

Above hypothesis determines trajectories of test particles. How about the dynamcis of the $g_{\mu\nu}$ themselves? This is the dynamics of Gravity.

Newtonian scheme insufficient because in it the gravitational force is instantaneously communicated over any physcial distance.

$$= \sum_{\mu\nu} g_{\mu\nu} \Delta x^{\mu} \Delta x^{\nu}$$

The coefficients $g_{\mu\nu}$ are called metric coefficients and give the scale of length and angle measurement but in General Relativity are dependent on space and time.

Above hypothesis determines trajectories of test particles. How about the dynamcis of the $g_{\mu\nu}$ themselves? This is the dynamics of Gravity.

Newtonian scheme insufficient because in it the gravitational force is instantaneously communicated over any physcial distance. *Violates Special Relativity*

Einstein needed a guiding principle to guess the Relativistic theory of Gravity, without any experimental data available at those speeds.

An analogy

In electromagnetism, we seek a principle to determine the dynamics of electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$. The sought after answer found by Lorentz is

$$S = \int d^4x(\mathbf{E} \cdot \mathbf{E} - \mathbf{B} \cdot \mathbf{B})$$

Out of the vector fields $\mathbf{E}$ and $\mathbf{B}$, we have to make up a quantity which is

1. A scalar under rotations, but equally importantly,

2. Invariant under Lorentz transformations

It can be shown that the above combination is the unique Lorentz invariant, and space-reflection invariant expression to quadratic order which can be made out of the $\mathbf{E}$ and the $\mathbf{B}$.

Thus, if we knew the Lorentz transformation properties of these fields, we need not have waited for two hundred years of experimentation, from Coulomb through Ampére, Faraday and finally the theoretical synthesis of Maxwell.

Einsteinian Dilemma

“The sought after generalisation will surely be of the form

$$\Gamma_{\mu\nu} = \kappa T_{\mu\nu},$$

where κ is a constant and $\Gamma_{\mu\nu}$ is a contravariant tensor of second rank that arises out of the fundamental tensor $g_{\mu\nu}$ through differential operations ... ...it proved impossible to find a differential expression for $\Gamma_{\mu\nu}$ that is a generalisation of [Poisson’s] $\nabla^2\phi$, and that is a tensor with respect to arbitrary transformations ... ... It seems most natural to demand that the system be covariant against arbitrary transformations. That stands in conflict with the result that the gravitational field does not possess this property."

[A. Einstein and M. Grossmann, 1913]

The Einstein-Hilbert Action Principle

While Einstein was unsure of what to put balance the two sides of the equations, Hilbert arrived at the answer, guided entirely by the transformatin properties of the quantites that must appear in the laws, namely, the components of the curvature tensor made out of the metric coefficients $g_{\mu\nu}$. His answer was,

$$S = \frac{1}{16\pi G} \int \sqrt{-\det g} d^4x \, R$$

The resulting equations are called Einstein's Equations, becasue he did find them on his own, using consistency arguments, directly at the level of the differential equations.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

The coefficient 8π is determined by demanding consistency with Newton's Law in the limit of weak gravitational fields.

Conclusion

- Cosmology has grown into a precise science

Conclusion

- Cosmology has grown into a precise science
- There are still outstanding mysteries arising from discrepancy of scales in fundamental theory and observed Universe

Conclusion

- Cosmology has grown into a precise science

- There are still outstanding mysteries arising from discrepancy of scales in fundamental theory and observed Universe

- These features probably point to new dynamics with newer find of energy/matter

- Or, they point to new laws of Physics

Conclusion

- Cosmology has grown into a precise science
- There are still outstanding mysteries arising from discrepancy of scales in fundamental theory and observed Universe
- These features probably point to new dynamics with newer find of energy/matter
- Or, they point to new laws of Physics

- Can the new laws be guessed by symmetry principles alone?

- Can the new laws be guessed by symmetry principles alone?
  Subject matter of Superstring Theory

The future awaits your particiation!