Unified Theories and Cosmology : the changing paradigm

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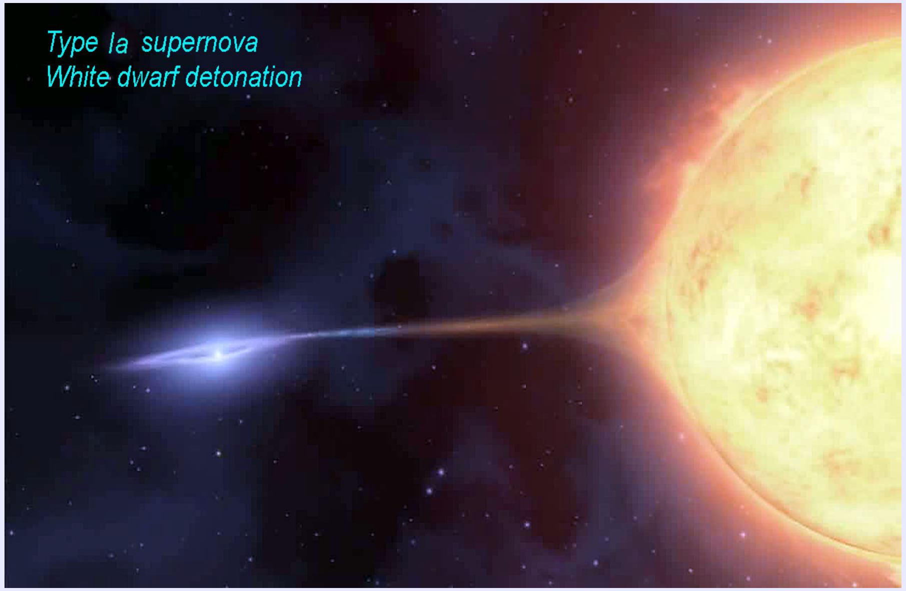
1 The latest news in press ...

Example of Supernova *Iax* discovered!!!

Excited? .... why should one be?

1.1 Universal feature of SN Ia

- A medium size progenitor of $\lesssim$8 solar mass has to relax to Chandrasekhar Limit 1.4 solar mass (Nobel 1983)
- In a binary system it may accrete more mass from the Red Giant companion
- Relaxation mechanism almost universal



Result : specific relation of absolute luminosity to timebase *Show Movie*

2 The expanding Universe

Proposed by Edwin Hubble 1929 – no Nobel –(: [The Hubble story]

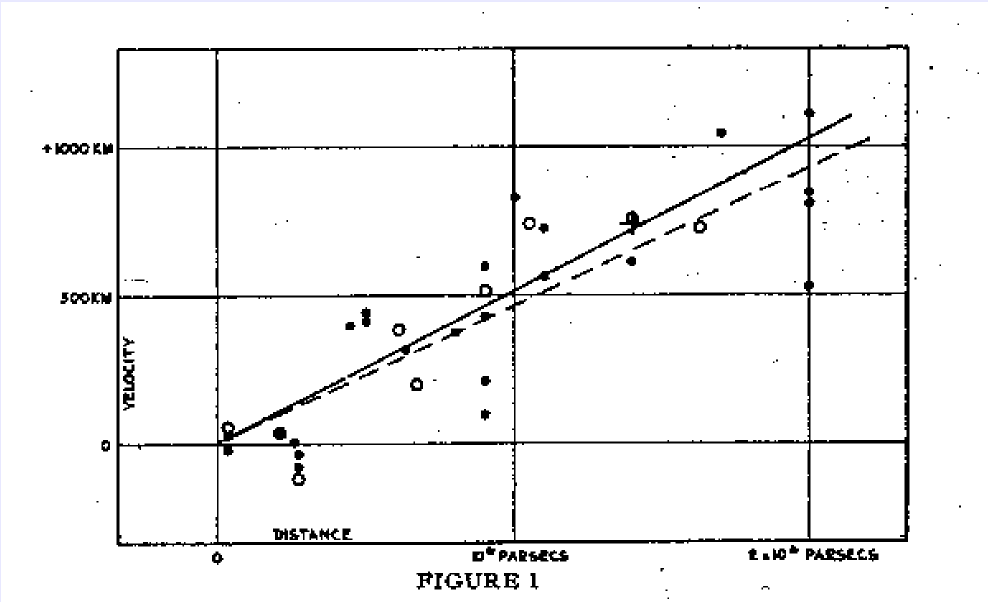
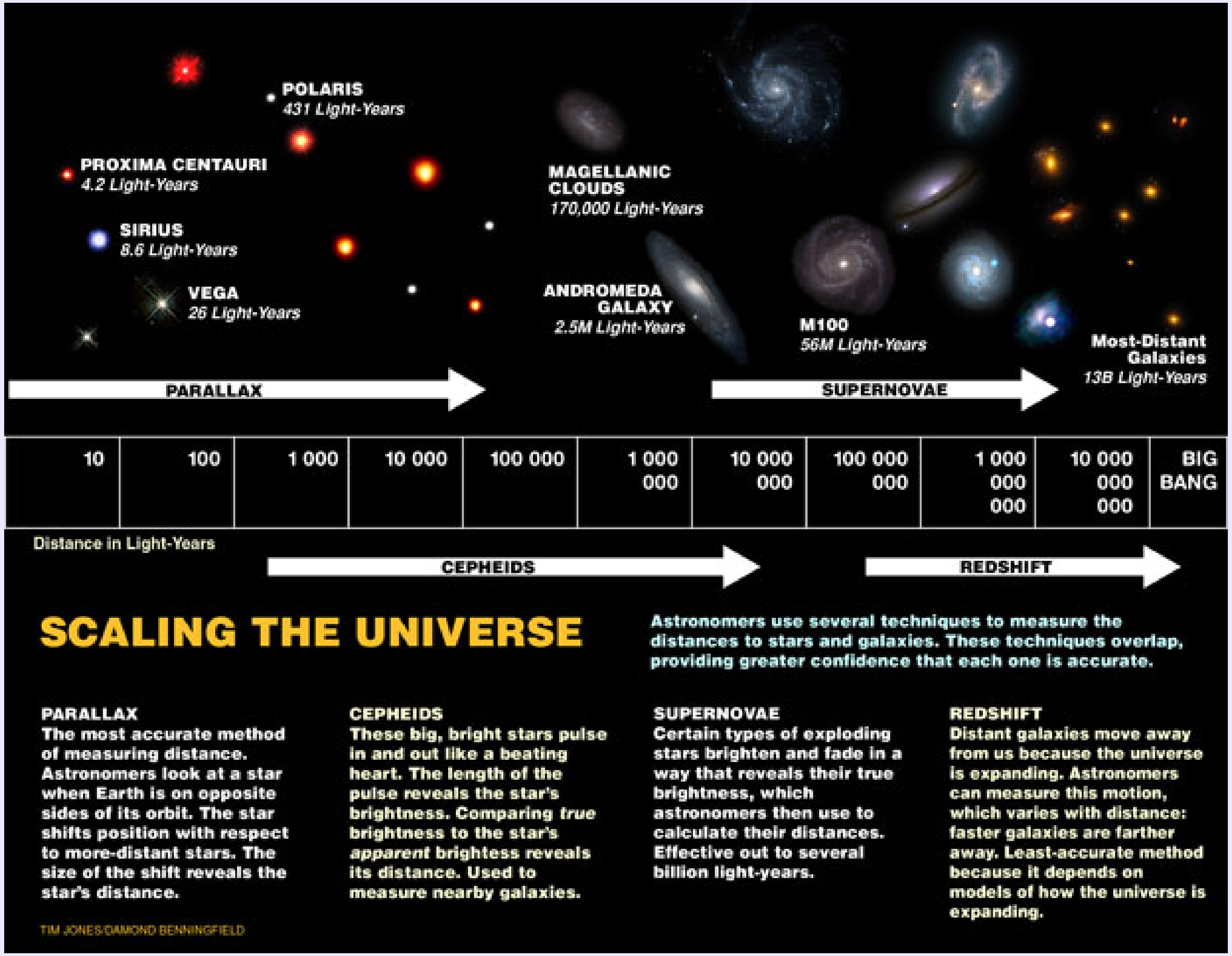


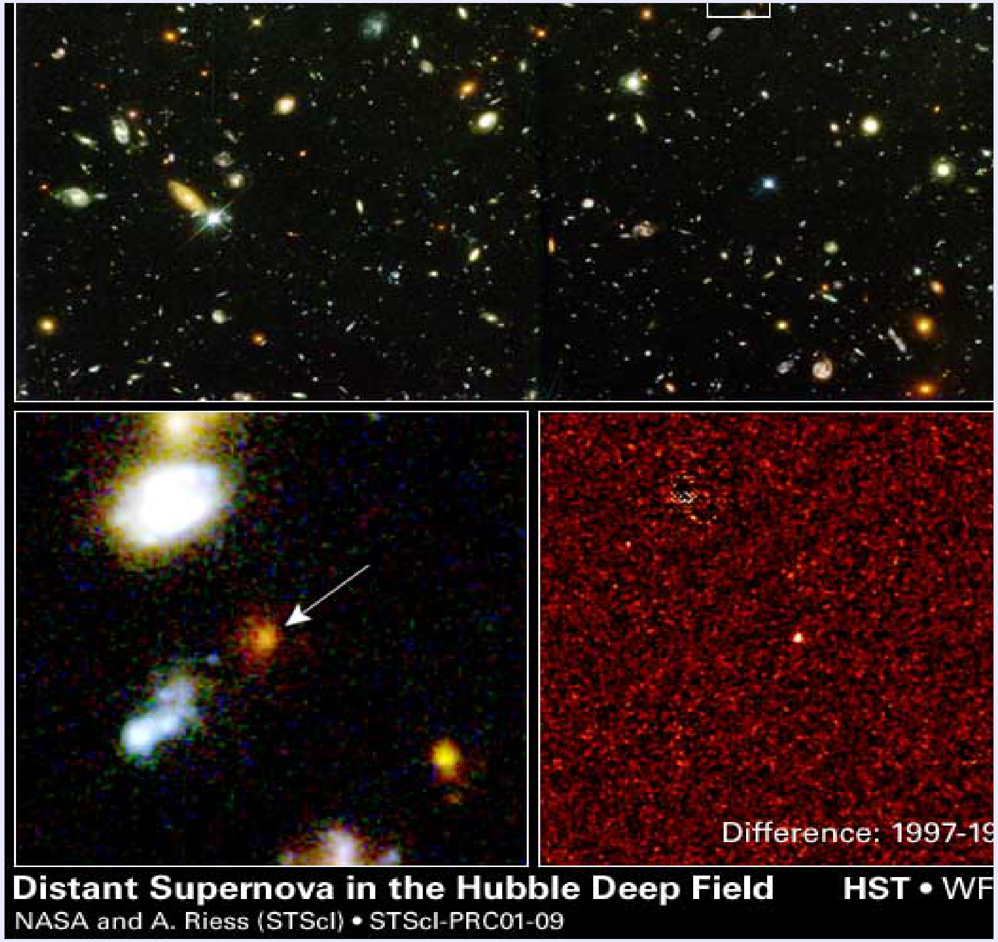
FIGURE 1

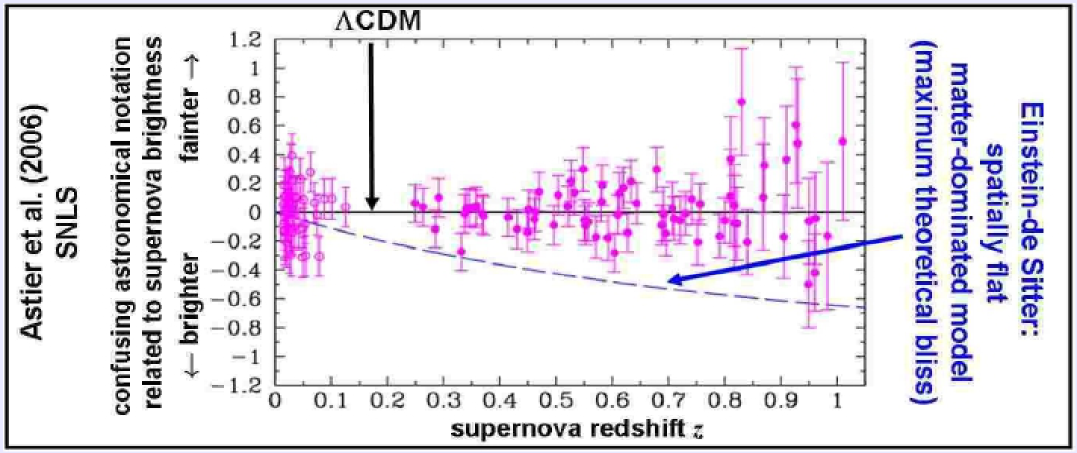
2.1 The need for "standard candles'

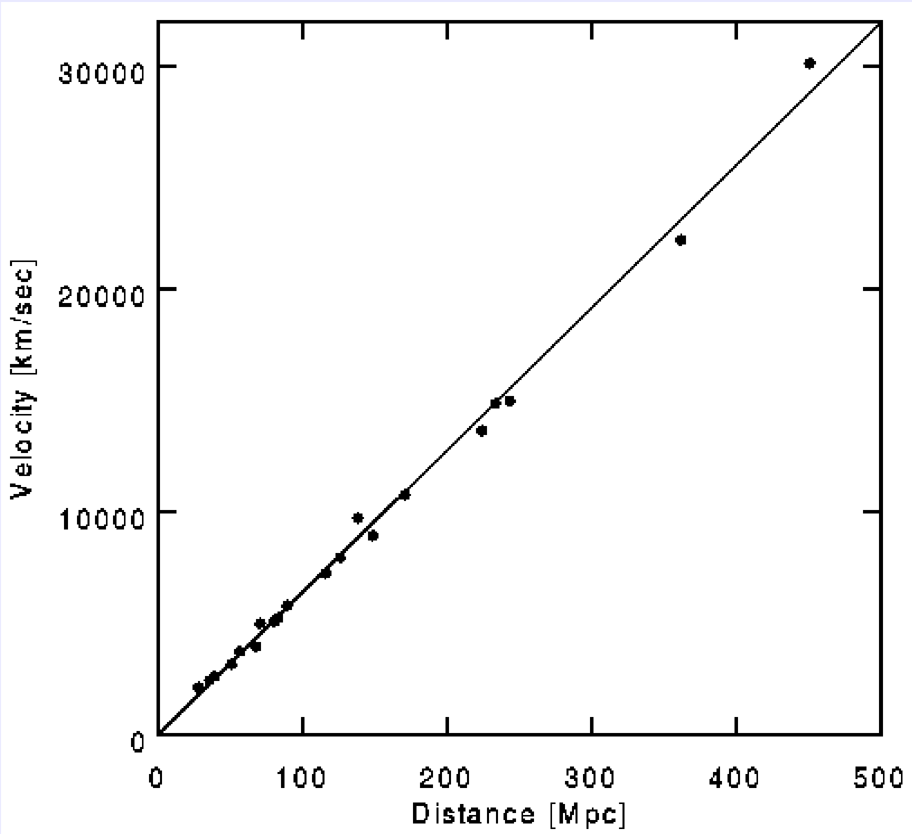
Calibrating cosmic distances

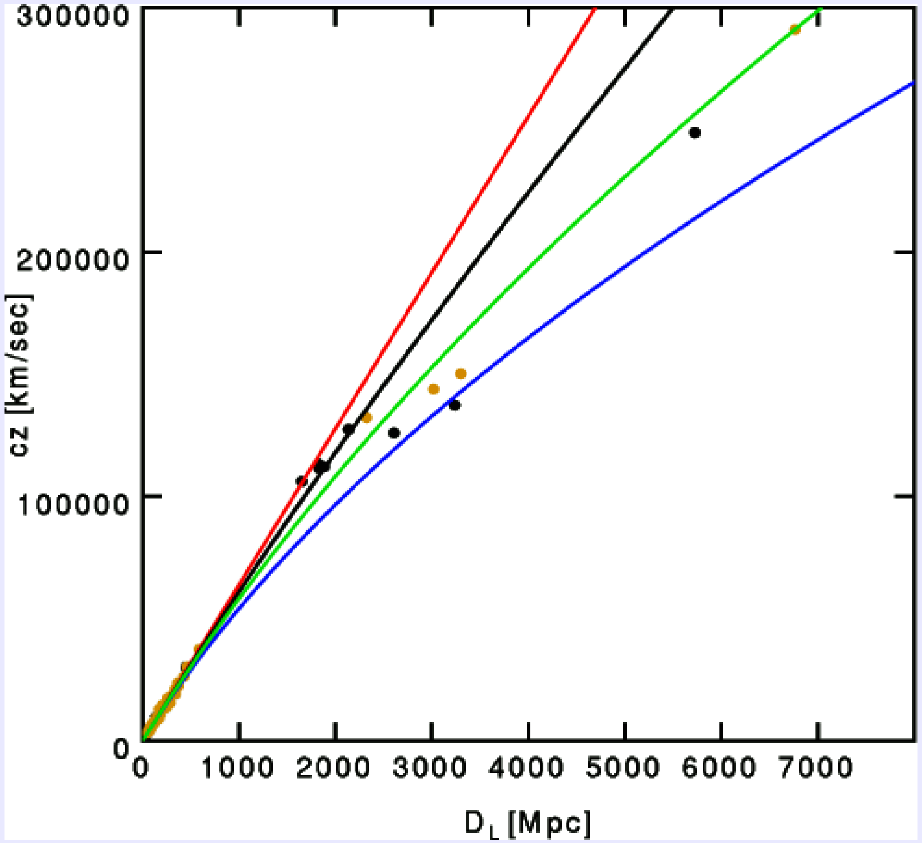


2.2 A blast from the remote past ... *Nobel 2011*




ΛCDM
Astier et al. (2006)
SNLS
confusing astronomical notation
related to supernova brightness
← brighter
fainter →
supernova redshift z
Einstein-de Sitter:
spatially flat
matter-dominated model
(maximum theoretical bliss)


Velocity [km/sec]
Distance [Mpc]


cz [km/sec]
D_L [Mpc]

2.3 An *accelerating* Universe??

- Einstein expected a static Universe
- Friedmann showed (1922) that General Theory of Relativity *required* an expanding Universe
  - it may or may not recollapse
- Hubble *discovered* the expanding Universe

Yet, the equations of GR expected only a decelerating Universe

$$\ddot{R(t)} = -\frac{4\pi}{3G}(\rho + 3p)$$

Heuristically this is exactly as expected of Gravity with ρ, p assumed >0.
But ... Relativistic principles allow energy-momentum of the form

$$T_{\mu\nu} = \Lambda\eta_{\mu\nu} \equiv \mathrm{Diag}(\rho \quad p \quad p \quad p) \Rightarrow p = -\rho$$

Unreasonable, (no such form of energy has been observed), but consistent.
This has come to called Dark Energy.

2.4 The laureates

Perlmutter, Riess, Schmidt

2.4.1 Type Iax

SN Type Iax has been identified as a progenitor of SN Ia

It can mimic some of the features of Ia but would have the wrong base line.

Foley et al (2012) :

There is a correlation between luminosity and light-curve shape, similar to that of SNe Ia, but offset from that of SNe Ia and with larger scatter.

Thus it is crucial to ensure this type in order to be sure of the calibration.

3 Plan of the talk

- The interface of Elementary Particles and Cosmology
- The Standard Model including the Higgs boson and its limitations
- What hints do we have for going past these obstacles :
  - Baryon asymmetry of the Universe
  - The possible discovery of the Right handed piece of the nuetrino

Will putting together these elements give us a unified theory?
Here it goes ...

4 Unified Theories – the geometry paradigm

- Gravity and Electricity shared the $1/r^2$ law
  - flux conservation $\Rightarrow$ charge conservation
- General relativity addresses the *kinematics vs dynamics* dichotomy of Newtonian schema
  - Conservation laws go for a toss but locality prevails
- Weyl's "gauge" principle $\psi(x) \to \psi(x)e^{ig\lambda(x)}$
- Isospin and the non-abelian gauge principle $\Psi(x) \to \Psi(x)e^{igT^a\lambda^a(x)}$
- Analogies, parallels ... but not unified.

5 The Standard Model : precursors

- Isotopic Spin (Heisenberg 1935)
  The strong force is independent of the charge on the nucleon.
  The Hamiltonina can be written in terms of the composite wave function

$$\Psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \text{ in analogy with spin wave function } \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix}$$

  $\psi_p \longrightarrow$ state of "Isotopic spin" $I_3 = \frac{1}{2}$

  $\psi_n \longrightarrow$ state of "Isotopic spin" $I_3 = -\frac{1}{2}$

- Exchange force / Intermediate vector boson (Yukawa 1935)
  [Figure]

5.1 How the world works – the covariant derivative

Mathematically the exchange force is specified by Covariant Derivative, which is generalisation of Minimal Coupling.

$$\vec{p} \to \vec{p} - \frac{q}{c}\vec{A}; \qquad H \to H - q\phi$$

which translated to Quantum Mechanics reads

$$\begin{aligned} -i\hbar\nabla &\to -i\hbar\nabla - \frac{q}{c}\vec{A} \\ i\hbar\frac{\partial}{\partial t} &\to i\hbar\frac{\partial}{\partial t} - qA^0 \end{aligned}$$

$$\frac{D}{Dx^\mu}\psi \equiv \frac{\partial}{\partial x^\mu}\psi + iqA_\mu$$

$$[\hbar] = [c] = 1$$

Amazing truth : All the four forces obey this same basic scheme.
The covariant derivative for gravity also involves space-time.
An interaction rule is stated as gemotric action of shifting or translation.

5.2 The Standard Model : enigmas

The gauge group is $\mathrm{SU}(3) \otimes \mathrm{SU}(2)_L \otimes U(1)_Y$

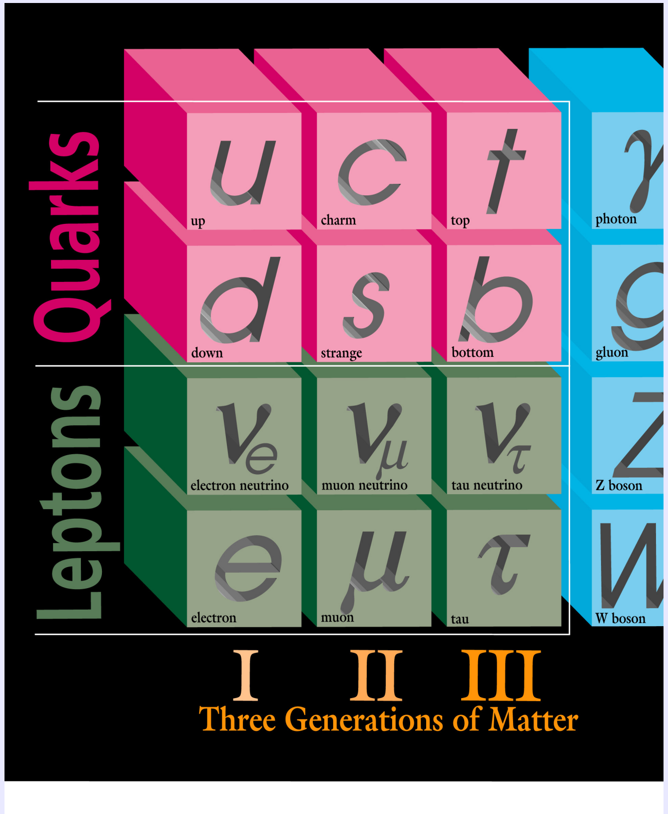
Left handed electron and (in 1967 the only known) left-handed neutrino are placed in a doublet Ψ_L of $\mathrm{SU}(2)_L$, while the right handed electron remains singlet under $\mathrm{SU}(2)_L$.

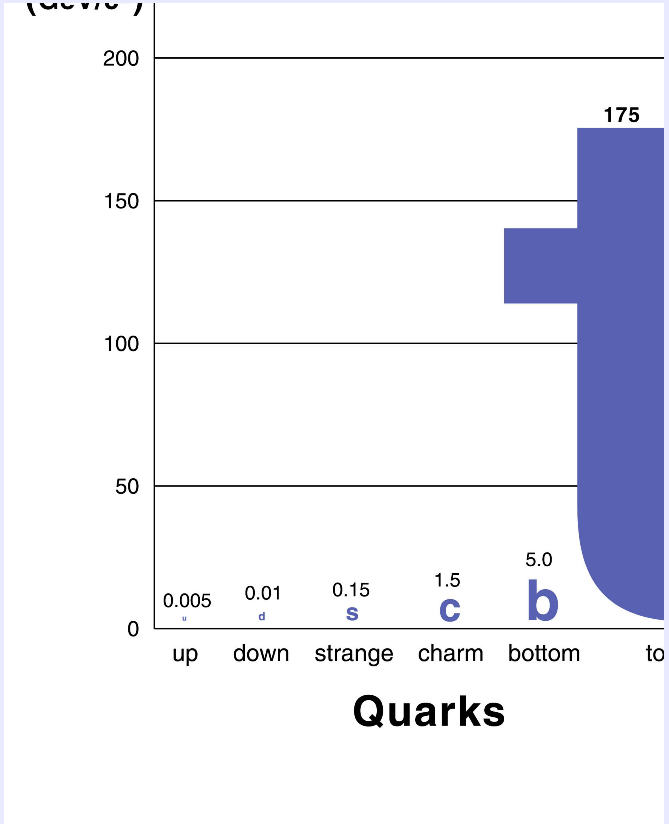
$$\begin{array}{cccc} & \tau_L^3 & \frac{1}{2}Y & Q \\ \begin{bmatrix} \nu_L \\ e_L^- \end{bmatrix} & \begin{array}{c} +\frac{1}{2} \\ -\frac{1}{2} \end{array} & \begin{array}{c} -\frac{1}{2} \\ -\frac{1}{2} \end{array} & \begin{array}{c} 0 \\ -1 \end{array} \\ e_R^- & 0 & -1 & -1 \end{array}$$

The building blocks of the Lagrangian are the covariant derivatives

$$D_\mu \Psi_L \equiv (\frac{\partial}{\partial x^\mu} + i g \tau^a W_\mu^a + i\,(-1) g' B_\mu) \Psi_L$$

$$D_\mu \psi_R \equiv (\frac{\partial}{\partial x^\mu} + i(-2)\, g' B_\mu) \psi_R$$





Enigmas :

– Three generations of fermions

– Fermion masses range from 1MeV to 157 GeV

– Higgs receives Planck scale 10^{19}GeV quantum corrections to its mass!!

5.3 Cosmology's oldest challenge

Matter-anti-matter asymmetry of the Universe

From Nucleosynthesis calculations and observed obundances of D, ^{3}He, ^{4}He and ^{7}Li,

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \cong 5 \times 10^{-10}; \qquad 0.017 < \Omega_B h^2 < 0.024$$
$$H_0 \equiv h100\,\mathrm{km}/s/\mathrm{Mpc}; \qquad h \cong 0.7$$

For comparison, random fluctuations at the QCD scale produce residual $\eta \sim 10^{-17}$. From WMAP data,

$$\Omega_B h^2 \cong 0.022$$

6 Genesis of baryogenesis

- CP violation discovery 1964
- CMBR discovery also 1965 ... expected from Big Bang
- The possibility of *explaining* baryon asymmetry as a dynamic effect (Weinberg Brandeis lectures 1965)
- Sakharov 1967 proposed a specific model

6.1 GUT scale baryogenesis

(Sakharov 1967; Yoshimura; Weinberg 1978)

1. There should exist baryon number B violating interaction

$$\begin{aligned} X \;\rightarrow\; & qq \qquad \Delta B_1 = \frac{2}{3} \\ & \bar{q}\bar{l} \qquad \Delta B_2 = -\frac{1}{3} \end{aligned}$$

2. Charge conjugation C must be violated

$$\mathcal{M}(X \rightarrow qq) \neq \mathcal{M}(\bar{X} \rightarrow \bar{q}\bar{q})$$

3. CP violation

$$r_1 = \frac{\Gamma(X \to qq)}{\Gamma_1 + \Gamma_2} \neq \frac{\bar{\Gamma}(\bar{X} \to \bar{q}\bar{q})}{\bar{\Gamma}_1 + \bar{\Gamma}_2} = \bar{r}_1$$

4. Out of equilibrium conditions
   Reverse reactions don't get the time to reverse the products

Net baryon asymmetry

$$\begin{aligned} B &= \Delta B_1 r_1 && + \Delta B_2(1 - r_1) \\ &\quad +(-\Delta B_1)\bar{r}_1 && + (-\Delta B_2)(1 - \bar{r}_1) \\ &= (\Delta B_1 - \Delta B_2)(r_1 - \bar{r}_1) \end{aligned}$$

- GUTs generically involve new gauge forces which mediate B violation
- Higgs scalar interactions can be natural source of CP violation
- The Particle Physics rates and expansion rate of the Universe compete

$$\Gamma_X \cong \alpha_X m_X^2/T; \qquad H \cong g_*^{1/2} T^2/M_{\rm Pl}$$

7 Parity and chirality

Dirac equation

$$i\gamma^0\frac{\partial\psi}{\partial t}+i\vec{\boldsymbol{\gamma}}\cdot\nabla\psi-m\psi=0$$

with the requirement that

$$(\gamma^0)^2=2I, \qquad \gamma^0\gamma^i+\gamma^i\gamma^0=0, \qquad \gamma^i\gamma^j+\gamma^j\gamma^i=-2I\delta^{ij}$$

A minimum four component equation and 4×4 γ- matrices are required. The Lagrangian density needed to obtain this equation is

$$\mathcal{L}=i\bar{\psi}\gamma^\mu\partial_\mu\psi-m\bar{\psi}\psi$$

But for $m=0$ we get two separate equations, the Weyl equations

$$i\frac{\partial\psi}{\partial t}-i\vec{\boldsymbol{\sigma}}\cdot\nabla\psi=0 \;\;\text{(eq.1), and}\;\; i\frac{\partial\psi}{\partial t}+i\vec{\boldsymbol{\sigma}}\cdot\nabla\psi=0 \;\text{(eq.2)}$$

7.1 Chirality

From the 4×4 γ-matrices we can construct the γ^5 matrix $i\gamma^0\gamma^1\gamma^2\gamma^3$ such that $\gamma^\mu\gamma^5 + \gamma^5\gamma^\mu = 0$.

We can now construct, given a 4-component fermion function ψ,

$$\psi_L = \frac{I - \gamma^5}{2}\psi \text{ and } \psi_R = \frac{I + \gamma^5}{2}\psi$$

We can check that ψ_L satisfies (eq.1) while ψ_R satisfies (eq.2)

Chirality is defined as the ratio $\frac{\mathrm{sgn}(\mathrm{energy})}{\mathrm{sgn}(\vec{S} \cdot \vec{p}\,)}$

Thus ψ_L contains left handed particles and right handed anti-particles
ψ_R contains right handed particles and left handed anti-particles

For $m = 0$, each doublet is a complete representation of Lorentz group.

Note that the mass term in the Dirac lagrangian is of the form

$$\mathcal{L}_{\mathrm{mass}} \sim m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L)$$

Thus both chiralities are needed to make up a massive spin-1/2 particle.

Madam Wu's experiment (1956) therefore meant that the neutrino was massless and that only one chirality had been singled out in nature.

(Sudarshan and Marshak; also noted by Feynman and Gell-Mann)

7.2 No mass terms permitted in Standard Model

Important automatic ingredient of SM construction :

$\overline{\psi_L}\psi_R$ is not gauge invariant

$\Leftarrow$ Left handed and right handed components have different gauge charges.

Solution : Introduce a scalar doublet field, the Higgs,
– with just the right charges,
– which allows interaction terms,
– which after symmetry breaking become effective mass terms

$$
\begin{aligned}
\mathcal{L}_{\text{Yuk}} &\sim h\,\overline{\Psi_L}\,\phi e_R^- \\
&\longrightarrow h\left(\;\bar{\nu}_L\;\overline{e_L^-}\;\right)\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}e_R^- \\
&\longrightarrow h\left(\;\bar{\nu}_L\;\overline{e_L^-}\;\right)\begin{pmatrix}0\\ v\end{pmatrix}e_R^-
\end{aligned}
$$

7.2.1 The virtue of chirality : from Revolution to Dogma

In SM, Chirality $\Rightarrow$have ones symmetry and get masses too, (in the broken symmetry phase).

Was Parity meant to be really lost???

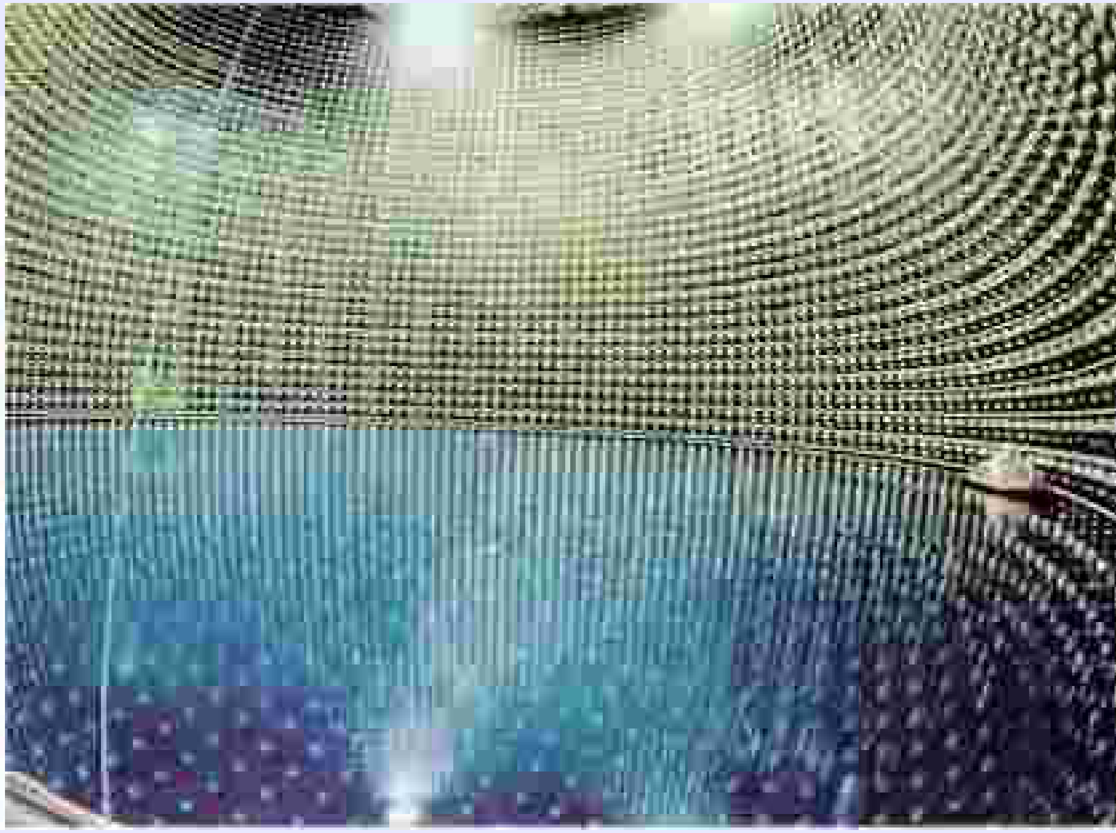
- Chirality is an elegant concept naturally embedded in the Quantum realistion of Lorentz Group –> spontaneous generation of mass.
- But that does not necessarily mean imbalance in Parity ie simple mirror reflection –> The world of qurks is both chiral and parity balanced.
- The observed P violation –> could be a spontaneously generated imbalance between right chiral and left chiral species
- This symmetry breaks in the early Universe
  - forming domains of opposite chiralities
  - the world we see is just one path our Universe took

8 Neutrino mass and after

Neutrino oscillations :

→ First glimpses by Davies 1973 – solar neutrino flux

→ Confirmed Super-Kamiokande 1998



8.1 The see-saw mechanism

Majorana vs. Dirac mass term

$$\psi_L^C \equiv \mathcal{C}\psi_L^*$$

$\rightarrow$ has the same transformation property under the Lorentz group, as ψ_R

$$\mathcal{L}_{\text{Maj}} \sim M\left(\overline{\psi_L^C}\psi_L + \overline{\psi_L}\,\psi_L^C\right)$$

The price we pay is that the fermion current is not conserved :

$$\frac{\partial}{\partial x^\mu}\left(\overline{\psi_L}\,\gamma^\mu\psi_L\right) = 2M\,\overline{\psi_L^C}\psi_L \text{ (check)}$$

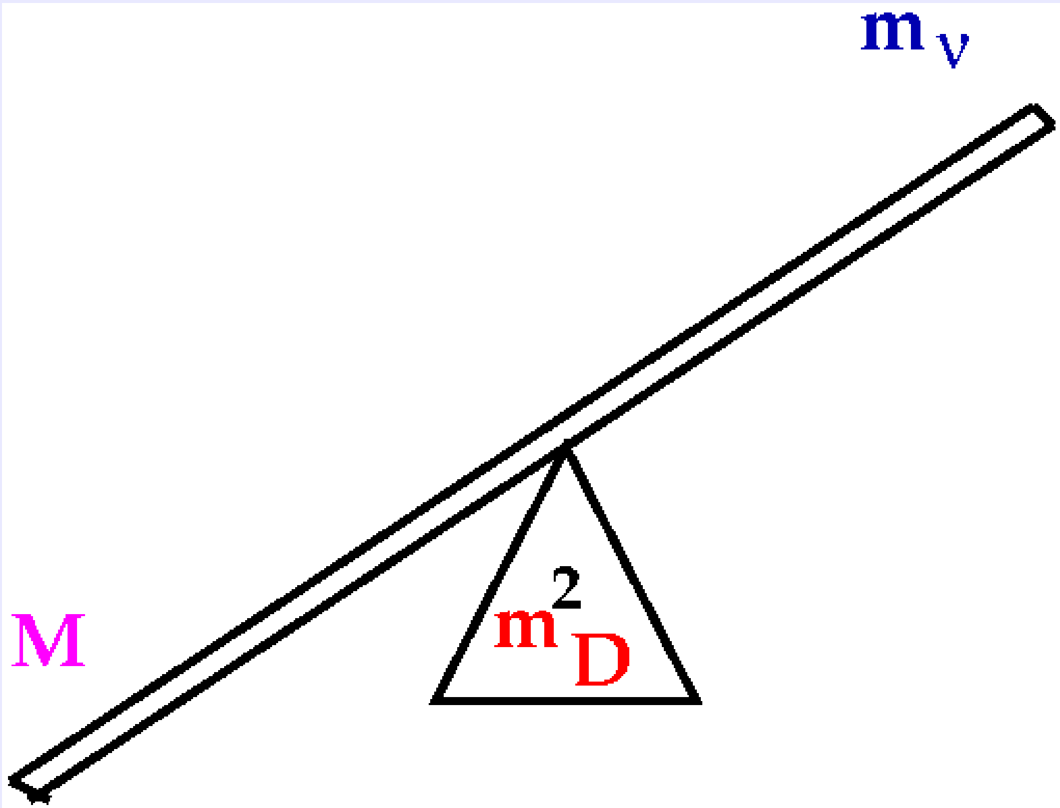
8.1.1 General mass matrix – the "see-saw" mechanism

In the general case where fermion number is not conserved (only neutrino sector so far), we need to diagonalise the mass matrix :

$$\begin{array}{cc} & \begin{array}{cc} \psi_L & \psi_R \end{array} \\ \begin{array}{c} \overline{\psi_L} \\ \overline{\psi_R} \end{array} & \begin{pmatrix} \mathcal{C}m_L & m_D \\ m_D & \mathcal{C}M_R \end{pmatrix} \end{array}$$

The relevance of the case $m_L \ll M_R$
(Gell-Mann, Ramond and Slansky 1978) :
The eigenvalues are

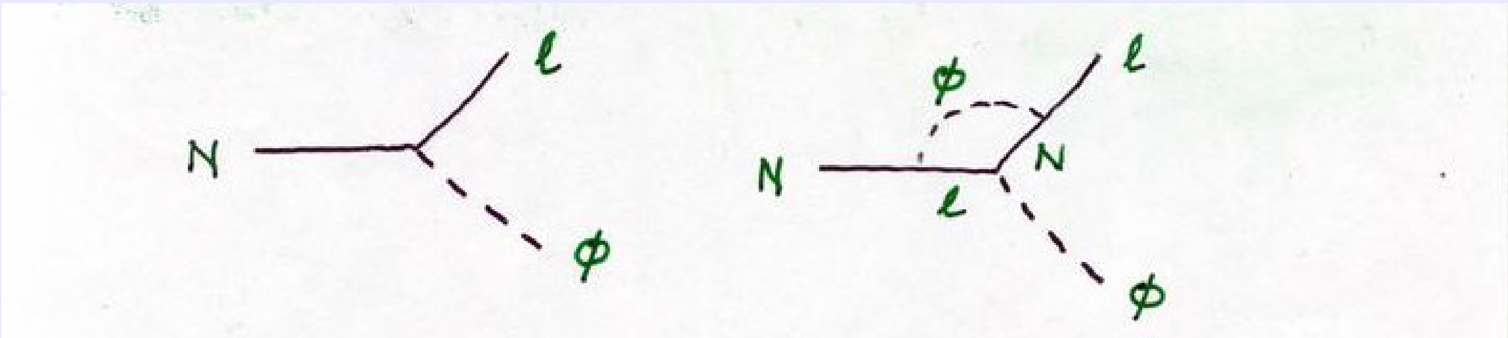
$$m_1 \simeq M_R; \qquad m_2 \simeq -\frac{m_D^2}{M_R}$$



8.2 Leptogenesis

(Fukugita and Yanagida 1986)

- Out of equilibrium decay of heavy Majorana neutrinos

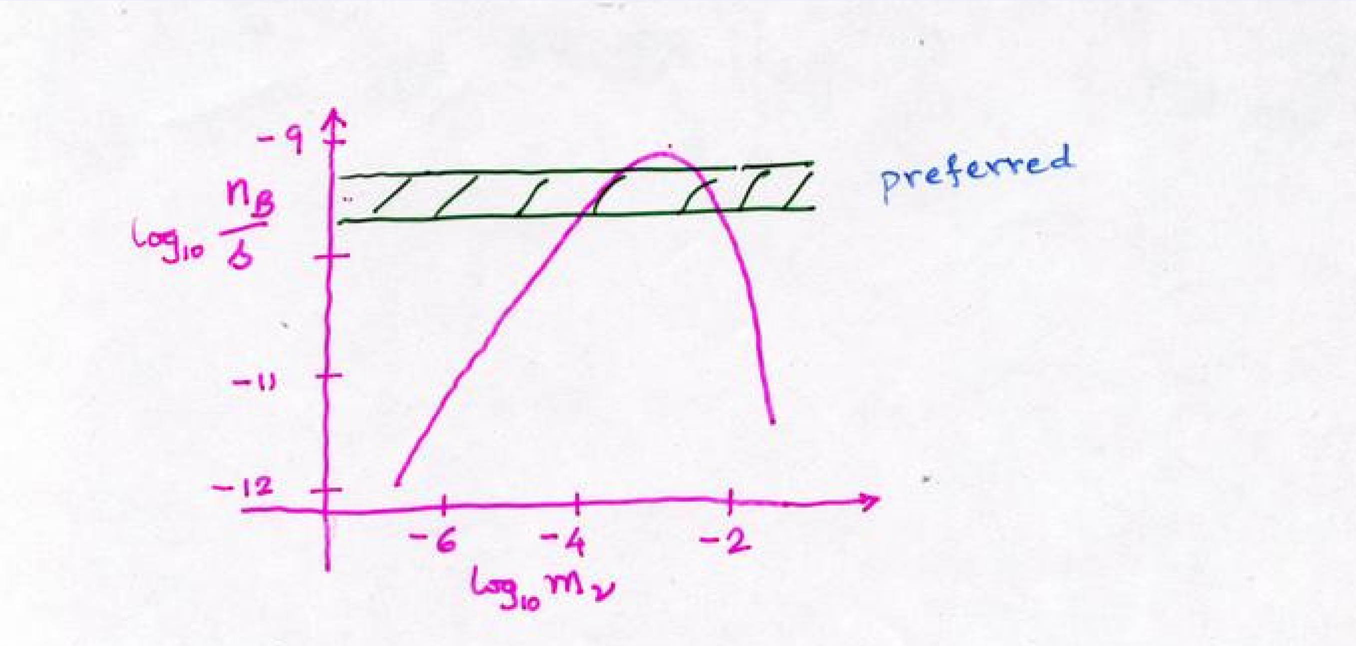


- Easy to arrange CP violation due to complex vacuum expectation values of scalar fields producing the mass

$$\frac{r-\bar{r}}{r} \sim \frac{1}{v^2 m_{_D}^2} \mathrm{Im}\left(m_{_D}^\dagger m_{_D}\right)^2$$

- Need to have comparable, faster, expansion rate of the Universe

Thermal leprogenesis in SO(10) (Buchmuller, Plumacher et al)

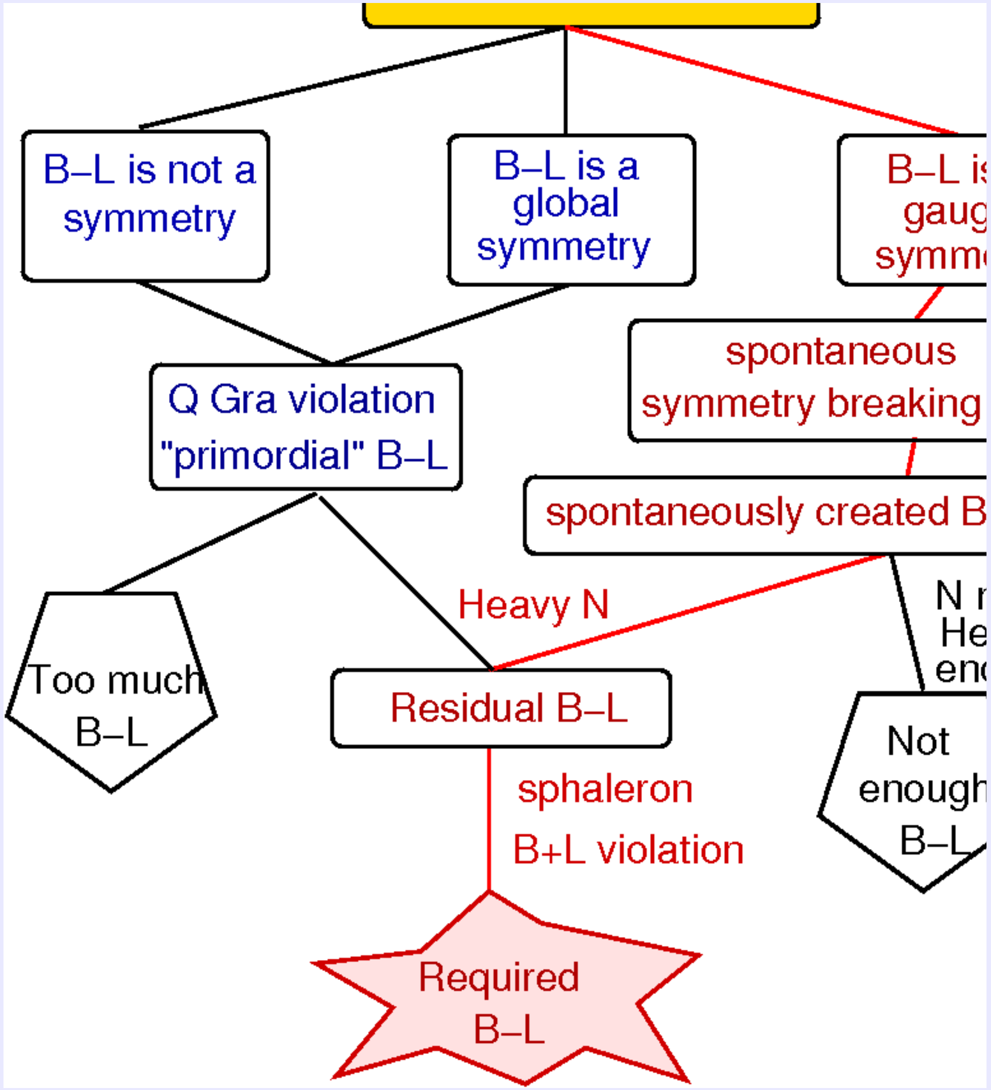


m_ν too small : Yukawa couplings too small to bring heavy N into equilibrium

m_ν too large : Erasure processes too efficient

$$M_N \gtrsim O(10^9)\mathrm{GeV}\left(\frac{2.5 \times 10^{-3}}{Y_N}\right)\left(\frac{0.05\mathrm{eV}}{m_\nu}\right)$$

8.2.1 What choices did Einstein's *der Alte* have?


B–L is not a symmetry
B–L is a global symmetry
B–L is gaug symm
spontaneous symmetry breaking
spontaneously created B
Q Gra violation "primordial" B–L
Heavy N
N r He enc
Too much B–L
Residual B–L
Not enough B–L
sphaleron
B+L violation
Required B–L

9 "Just" Beyond the SM ?

GUT naturalness of gauge coupling unification; —> see-saw M_N was expected to fit in.

- $\rightarrow$ It did, provided $m_D \approx 100$GeV. (Still $M_{\text{GUT}} \sim 10^{16}$GeV, $M_N \sim 10^{12}$GeV)
- $\rightarrow$ The only guide to neutrino Dirac mass m_D could be charged fermions mass.
- $\rightarrow$ Unfortunately m_D values for charged fermions are scattered from 175GeV to 1 MeV.
- $\rightarrow$ Unfortunately also, light neutrino mass differences (known since 1998) imply an order of magnitude variation in m_2 values.

"Beyond Standard Model efects only at high scale" is a prejudice !!

10 Left-right as JBSM

Just Beyond the Standard Model ... $\mathrm{SU}(3) \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes U(1)_X$

$$
\begin{array}{cccccc}
 & \tau_L^3 & \tau_R^3 & \frac{1}{2}X & Q \\
\lceil \; \nu_L \; \rceil & +\frac{1}{2} & 0 & -\frac{1}{2} & 0 \\
\lfloor \; e_L^- \; \rfloor & -\frac{1}{2} & 0 & -\frac{1}{2} & -1 \\
\lceil \; \nu_R \; \rceil & 0 & +\frac{1}{2} & -\frac{1}{2} & 0 \\
\lfloor \; e_R^- \; \rfloor & 0 & -\frac{1}{2} & -\frac{1}{2} & -1
\end{array}
$$

$$
\begin{array}{cccccc}
 & \tau_L^3 & \tau_R^3 & \frac{1}{2}X & Q \\
\lceil \; u_L \; \rceil & +\frac{1}{2} & 0 & +\frac{1}{6} & +\frac{2}{3} \\
\lfloor \; d_L \; \rfloor & -\frac{1}{2} & 0 & +\frac{1}{6} & -\frac{1}{3} \\
\lceil \; u_R \; \rceil & 0 & +\frac{1}{2} & +\frac{1}{6} & +\frac{2}{3} \\
\lfloor \; d_R \; \rfloor & 0 & -\frac{1}{2} & +\frac{1}{6} & -\frac{1}{3}
\end{array}
$$

10.1 Gauged $B - L$

- Introduced new species ν_R as a partner to e^-_R
- New gauge symmetry $\mathrm{SU}(2)_R$
- Need a new hypercharge $X \rightarrow$ turns out to be exactly $B - L$
- In praise of $B - L$ ... the only conserved charge of SM which is not gauged! $\rightarrow$ Hereby it gains the status of being gauged

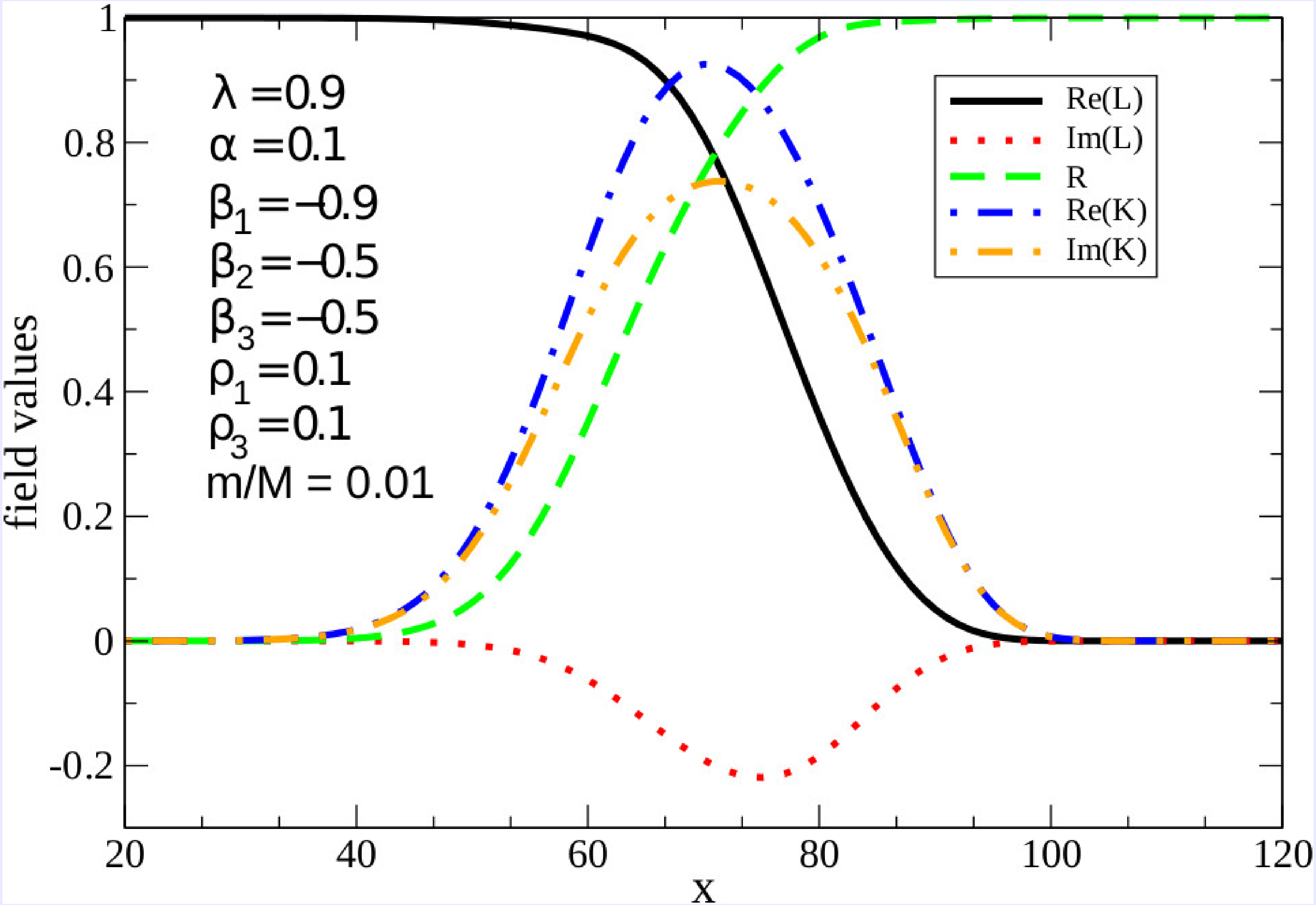
10.2 Non-thermal leptogenesis

If we ask the reverse question : if the N mass is not as high as required for thermal Leptogenesis, do we still have the scope for producing baryon asymmetry?

The answer is yes. (Sarkar, UAY 2007)

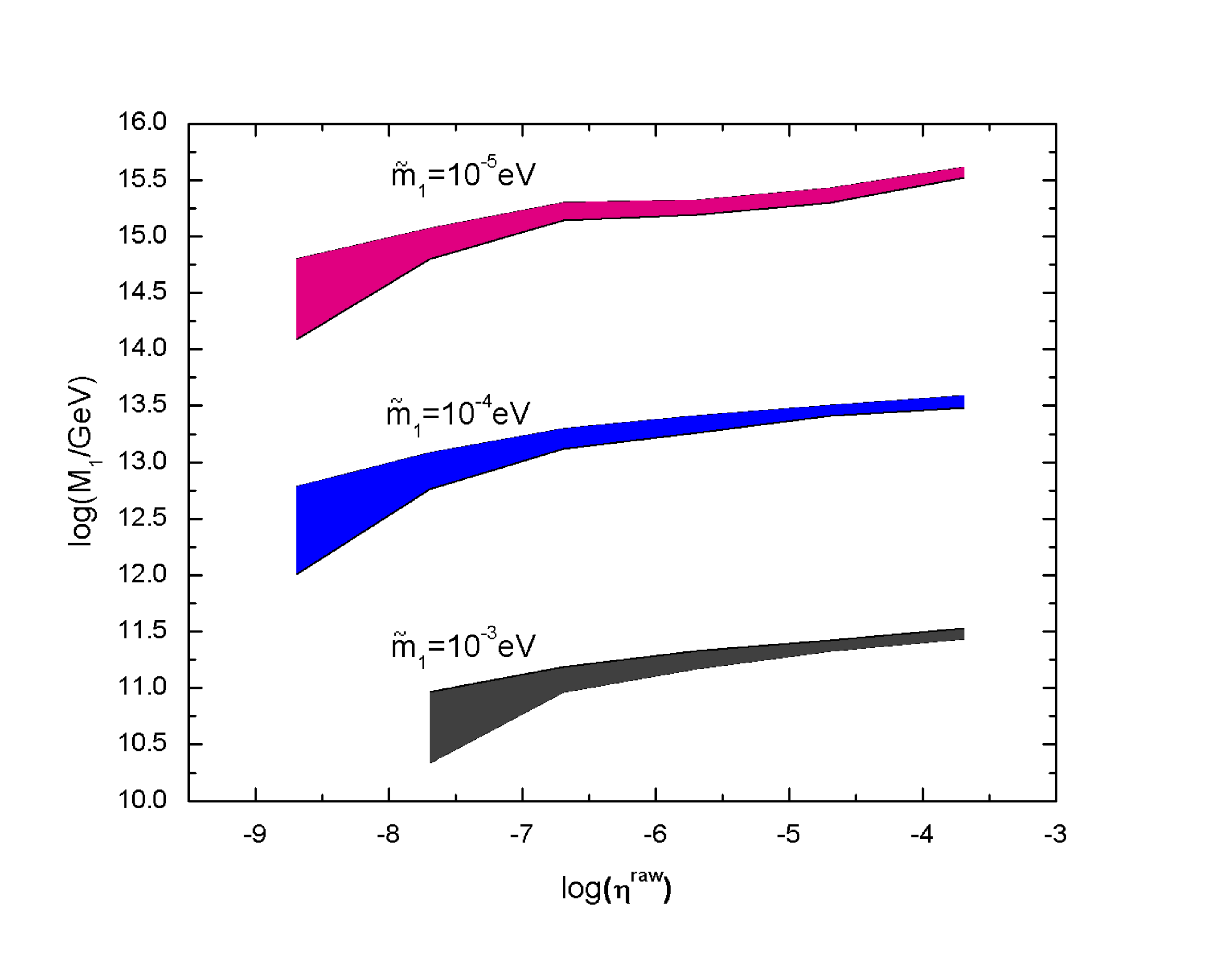
- The left-right symmetric model has domain walls, with sufficient CP violation provided by the scalar condensates to produce lepton number at a low scale.

- The effect is the same as having bubble walls

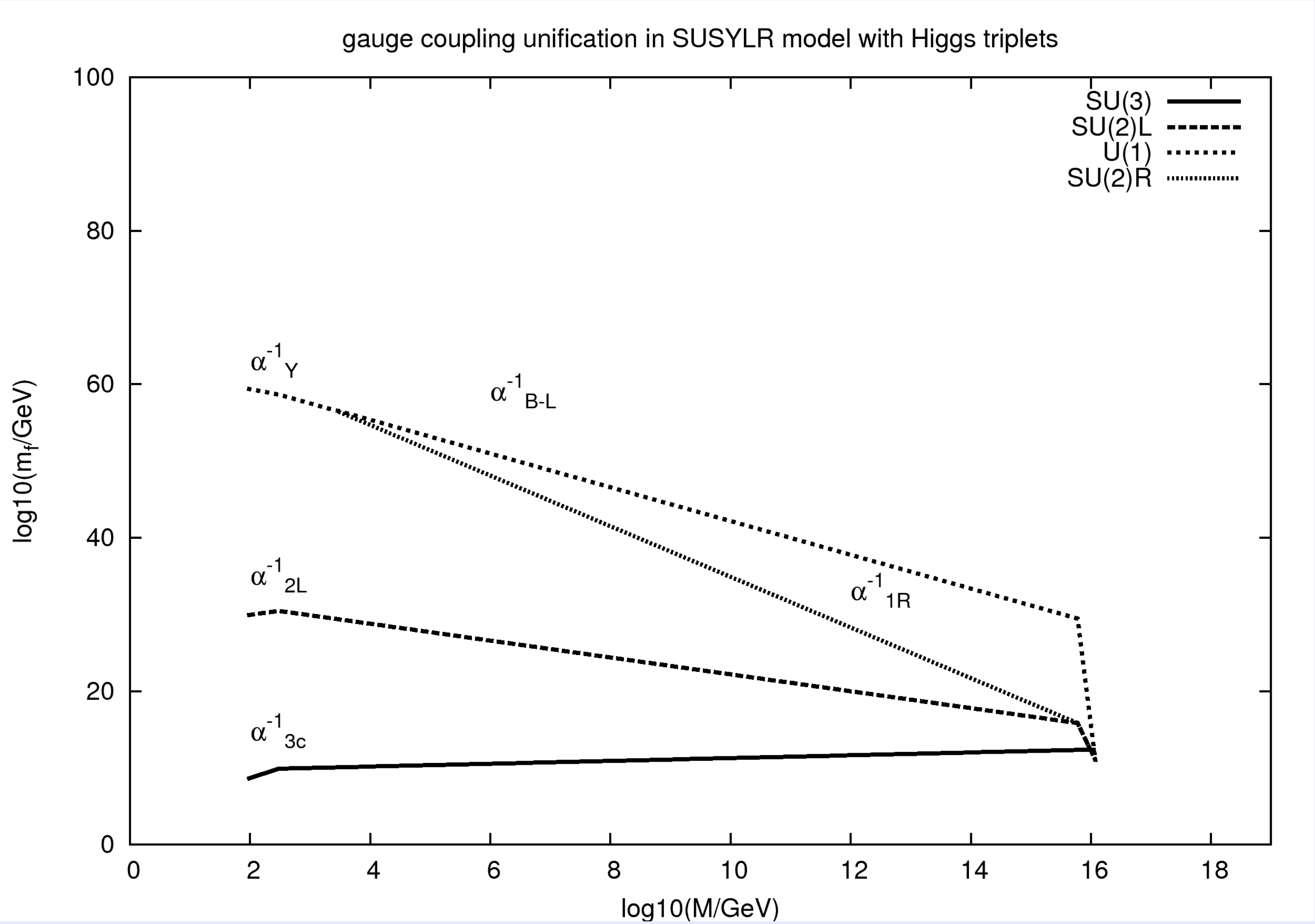


Can this lepton asymmetry survive?

This question was answered in the affirative, solving Boltmann equtions (Narendra Sahu and UAY 2005)



10.2.1 Unification of couplings & scale of $B-L$ violation



3TeV $U(1)_{B-L}$! (Debasish Borah, UAY 2012)

11 Conclusions and caveats

- Conceptual unification – yet many coupling constants
- Cosmology of relics is a miser on information
- Our recommendation : Believe in JBSM Left-Right model
  - UV completion through SUSY / extra dimensions
  - Leptogenesis (Narendra Sahu) through L-R domain walls –> Unambiguous nature of the phase transition
  - Domain walls decay (Anjishnu Sarkar, Sasmita Mishra) may be found in gravitational waves
  - Low scale $B - L$ (Debasish Borah) may be found in colliders / $N \leftrightarrow \bar{N}$ oscillations.

THANK YOU