Unification, Cosmology … and all that

U A Yajnik

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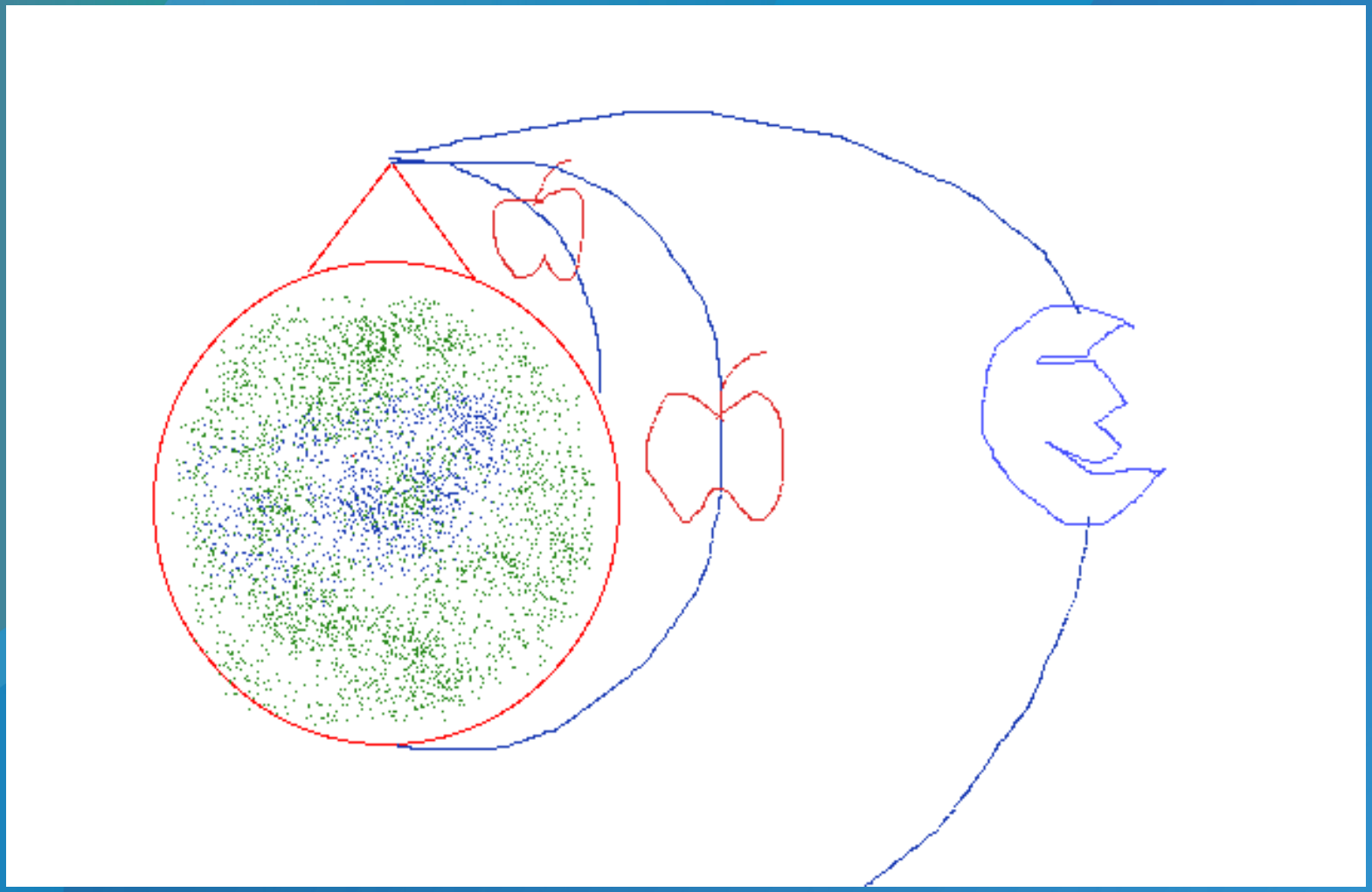
Triumphs and Travails of *theoretical Physics*

- Energy which is not energy
- Symmetries that are not symmetries
- Constants that are not constants
- S-matrix that is not S-matrix

Part I
Welcome to Geometry, Farewell to Energy

Newton unites the heavens and the earth



Universal dynamics of apples and planets

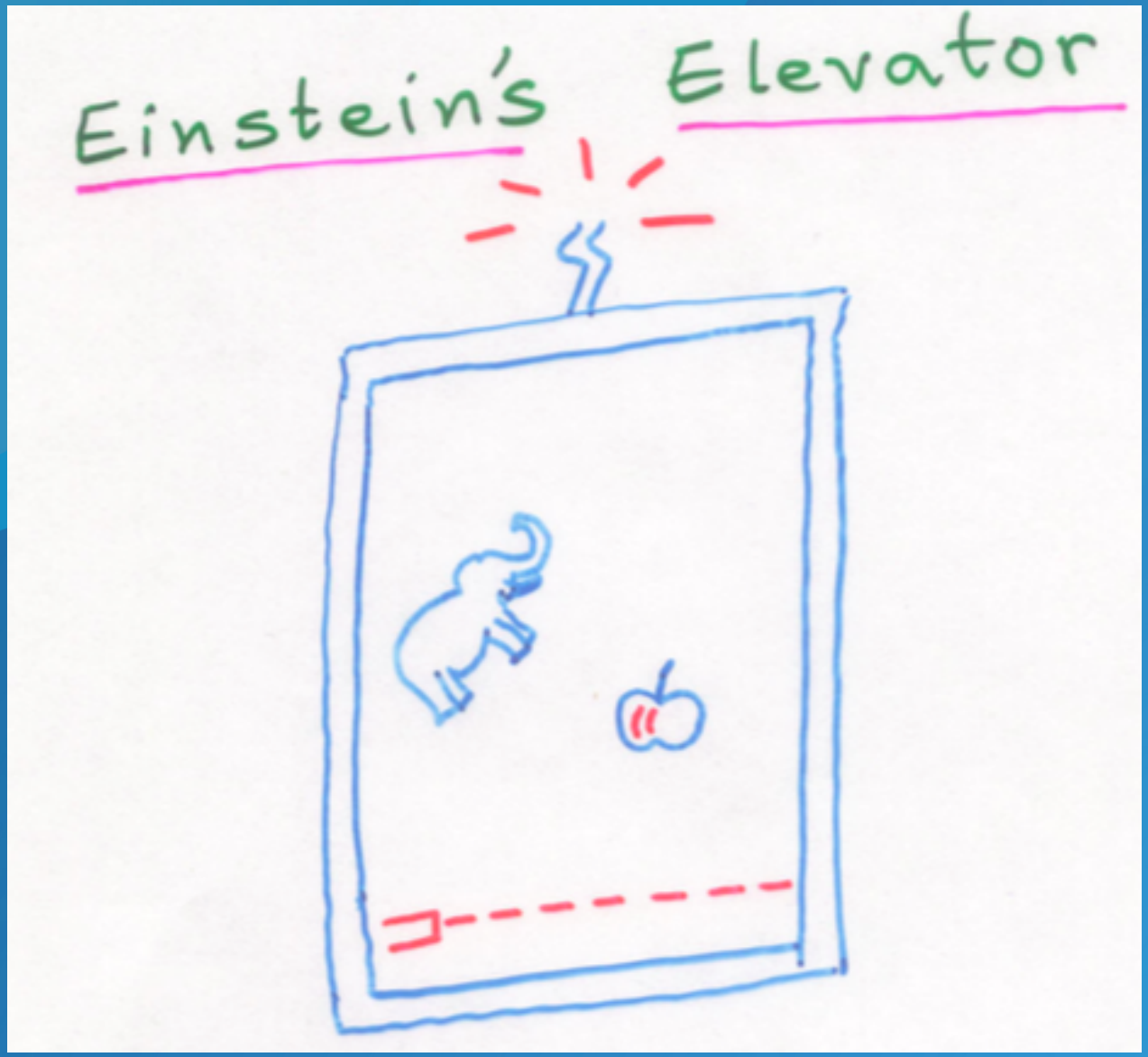
… *the "firmament" fathomed*

Einstein resolves *the great dichotomy*

- Newton's dichotomic schema
  - Law of dynamics $F = ma$
  - Law of force $F = \alpha \cdot f(r_i \ldots)$
- Principle of Equivalence
- Mathematical : "General Covariance"
  - General curvilinear coordinate transformations
- – Dynamically determined space-time metric

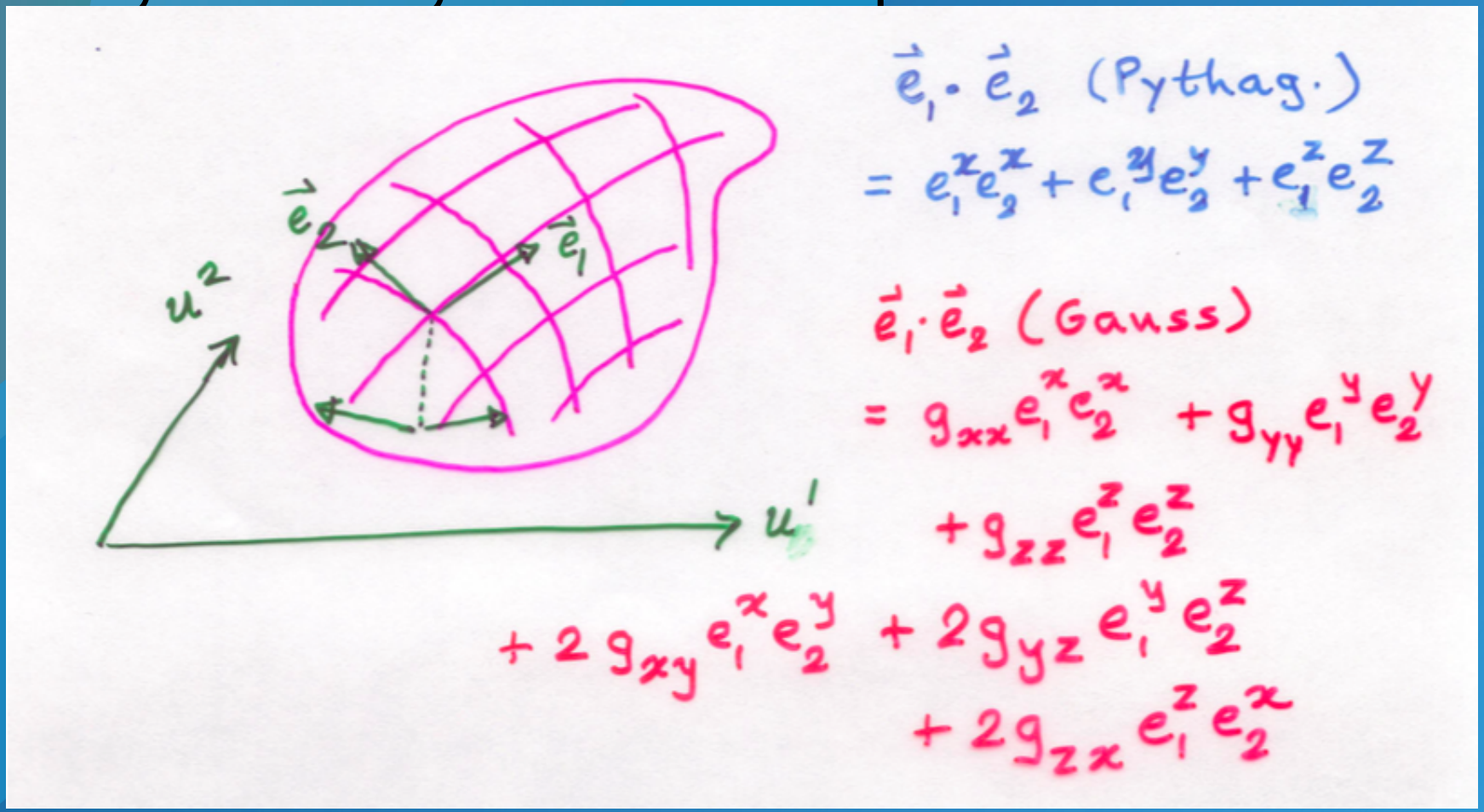
Einstein resolves ….. *dichotomy*

- Gravity is not a “force”
- Paths are geodesics of the geometry
- Do away with “force” if gravity is the only kind of interaction.



Einstein resolves … *dichotomy*

- Equivalence of all frames : “General Covariance”
  - General curvilinear coordinate transformations
  - Dynamically determined space-time metric



Einstein's schema

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=8\pi G\,T_{\mu\nu}$$

$$\frac{d^2x^{\mu}(\tau)}{d\tau^2}+\Gamma^{\mu}_{\nu\rho}\frac{dx^{\nu}}{d\tau}\frac{dx^{\rho}}{d\tau}=0$$

(Derived from above)

(An identity)

$$\frac{\partial}{\partial x^{\mu}}T^{\mu\nu}-\Gamma^{\nu}_{\mu\rho}T^{\mu\rho}-\Gamma^{\mu}_{\mu\rho}T^{\nu\rho}=0$$

- Where $R_{\mu\nu}\sim\partial\partial g,\partial g\partial g$ and $\Gamma\sim\partial g$
- "Gravity tells matter how to move"
- "Matter tells gravity how to curve"
- No force, no inertia. Particles follow geodesics.

Einstein's schema – farewell to conserved Energy

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$$

which is

(i) covariant and

(ii) obeys the vanishing of its covariant divergence.

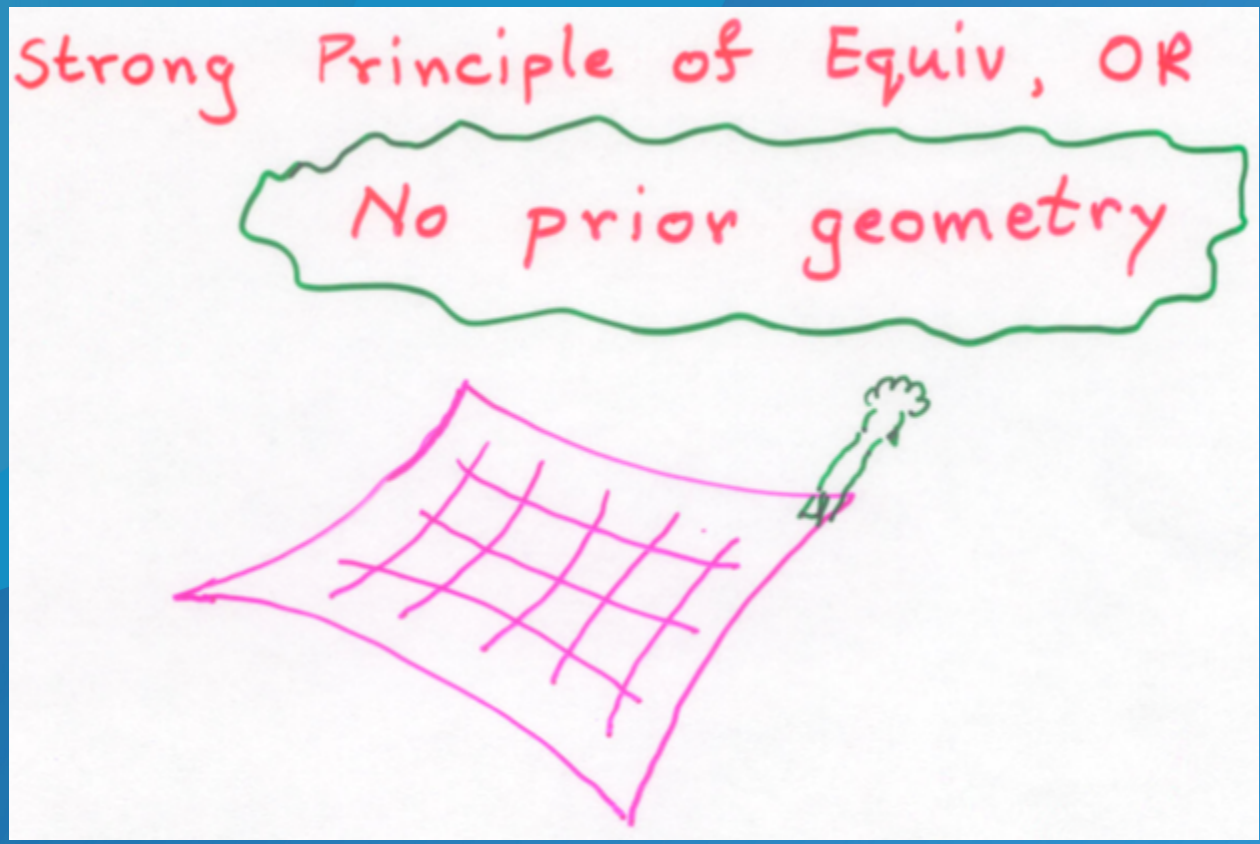
Correspondingly for the energy-momentum tensor $T_{\mu\nu}$ we get covariantly vanishing divergence and no conserved current

$$\frac{\partial}{\partial x^{\mu}} T^{\mu\nu} - \Gamma^{\nu}_{\mu\rho} T^{\mu\rho} - \Gamma^{\mu}_{\mu\rho} T^{\nu\rho} = 0$$

Einstein resolves …at what cost *??!!*

- Principle of Equivalence also determines the dynamics of Gravity itself

- The equations for the metric of the spacetime also in turn determine the global topology of the space time!!



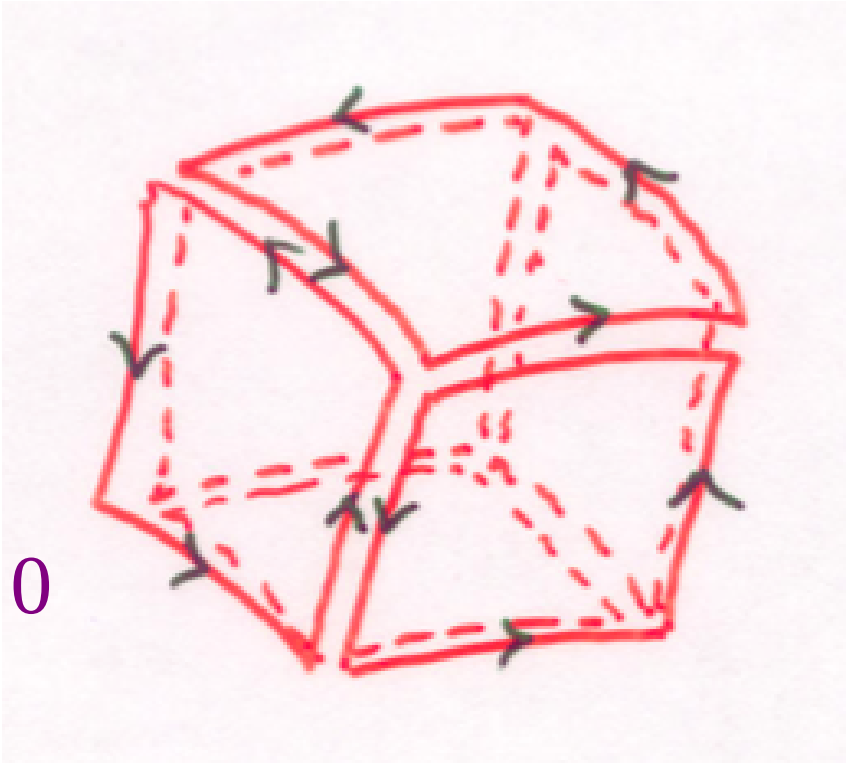
- Thus we stare into an abyss
- Singular solutions like the interior of a black hole or the Big Bang.
- Here even the basic scaffolding of Physics, spacetime breaks down

Einstein resolves … dichotomy

Lost in Physics, recovered in Geometry

"Covariant conservation" of energy density follows from Bianchi identities

$$\frac{D\,R_{\lambda\mu\nu\rho}}{D\,x^{\sigma}}+\frac{D\,R_{\lambda\mu\rho\sigma}}{D\,x^{\nu}}+\frac{D\,R_{\lambda\mu\sigma\nu}}{D\,x^{\rho}}=0$$



"Boundary of a boundary = 0"

Part II
“Gauge Symmetry” the arbiter of interactions

Isotopic spin – a “true” symmetry

- Heisenberg proposes an abstract “spin”

$$\Psi(x)=\begin{pmatrix}\psi_p(x)\\ \psi_n(x)\end{pmatrix}$$

- A symmetry of the Strong nuclear force

$$H=\frac{\hbar^2}{2M}\nabla\Psi^*\nabla\Psi+a_1\Psi^*\Psi+\ldots$$

- Invariance under

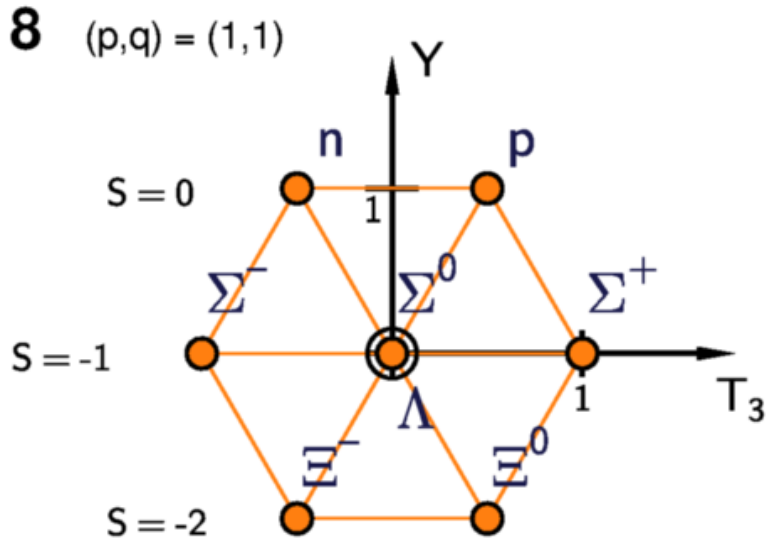
  (i) Overall phasing $\Psi(x)\rightarrow e^{i\lambda}\Psi(x)$ Conserved B

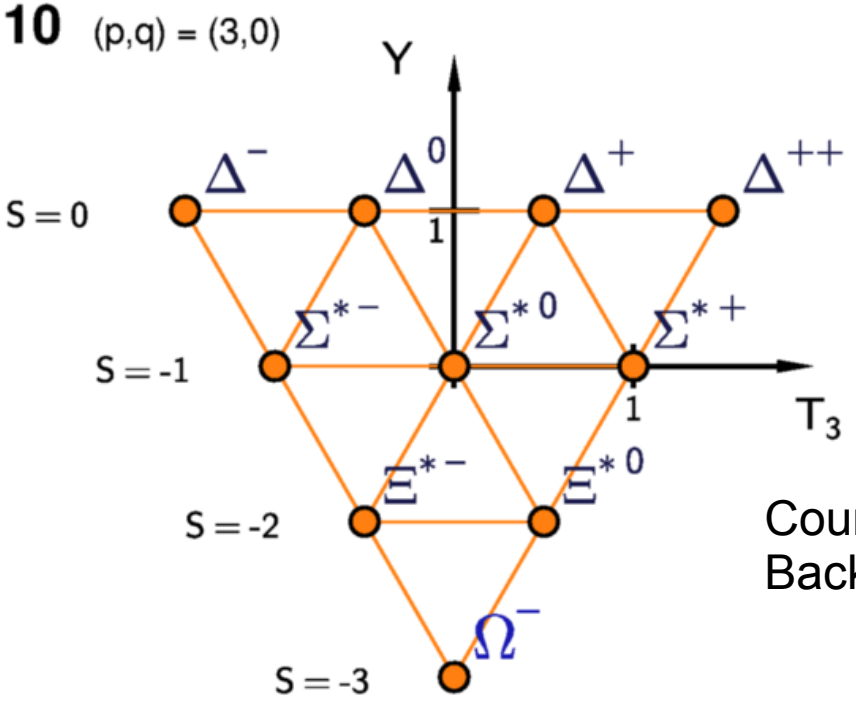
  (ii) Isospin rotations $\Psi(x)\rightarrow e^{i\tau^a\lambda^a}\Psi(x)$ Conserved τ^3

Hadron spectroscopy

- SU(3) symmetry

  (later called flavour)
- Sakata, Ne'eman, Gell-Mann
- "Eightfold way"





Courtesy Backreaction

Weyl's gauge invariance

- Lorentz force in Hamiltonian formalism

$$H=\frac{1}{2m}\left(\vec{p}-\frac{q}{c}\vec{A}\right)^2+qA^0$$

- In Quantum Mechanics

$$i\hbar\left(\frac{\partial}{\partial t}+i\frac{q}{\hbar}A^0\right)\psi=\frac{\hbar^2}{2M}\left(\nabla-i\frac{q}{\hbar c}\vec{A}\right)\cdot\left(\nabla+i\frac{q}{\hbar c}\vec{A}\right)\psi$$

- Invariance under Local rephasing

$$\psi(x)\rightarrow e^{iq\lambda(x)/\hbar c}\psi(x)$$

$$A^0(x)\rightarrow A^0-\frac{1}{c}\frac{\partial}{\partial t}\lambda(x) \qquad \vec{A}(x)\rightarrow\vec{A}(x)+\nabla\lambda(x)$$

- This only gives a prescription for coupling to Electromagnetism, no new conserved charge

General Covariance and gauge invariance

- Local Lorentz invariance
- Covariant derivative

$$\frac{DV^{\nu}(x)}{Dx^{\mu}} \equiv \frac{\partial V^{\nu}(x)}{\partial x^{\mu}} + \Gamma^{\nu}_{\mu\rho} V^{\rho}(x)$$

- Metric tensor exists, and is “preserved”

$$\frac{Dg_{\mu\nu}(x)}{Dx^{\rho}} = 0$$

- Local phase invariance
- Covariant derivative

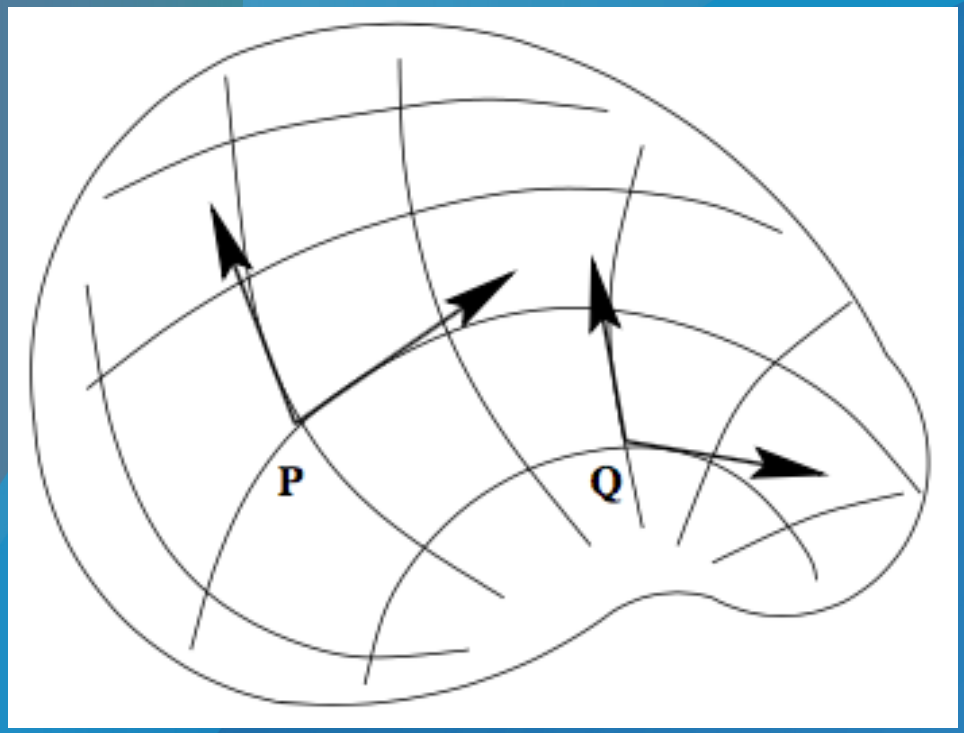
$$\frac{D\psi(x)}{Dx^{\mu}} \equiv \frac{\partial \psi(x)}{\partial x^{\mu}} + i\frac{q}{\hbar c} A_{\mu}(x)\psi(x)$$

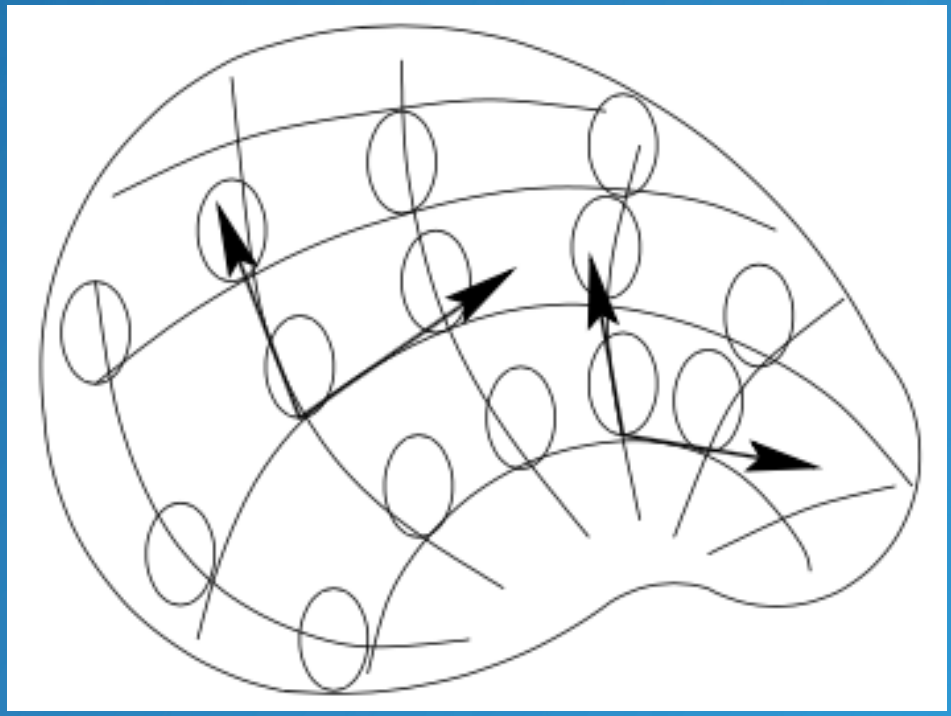
- No metric, only “connection”

General Covariance and gauge invariance – Local symmetry groups

Curvilinear Coords
= Local Lorentz transformation

Gauge invariance
= Local U(1) invariance
(Kaluza's fifth dimension)





Gauge Invariance for isospin?

- Combine Weyl and Heisenberg?

$$\Psi(x) \equiv \begin{pmatrix} \psi_p(x) \\ \psi_n(x) \end{pmatrix} \rightarrow \exp\{ i \tau^a \lambda^a(x) \} \Psi(x)$$

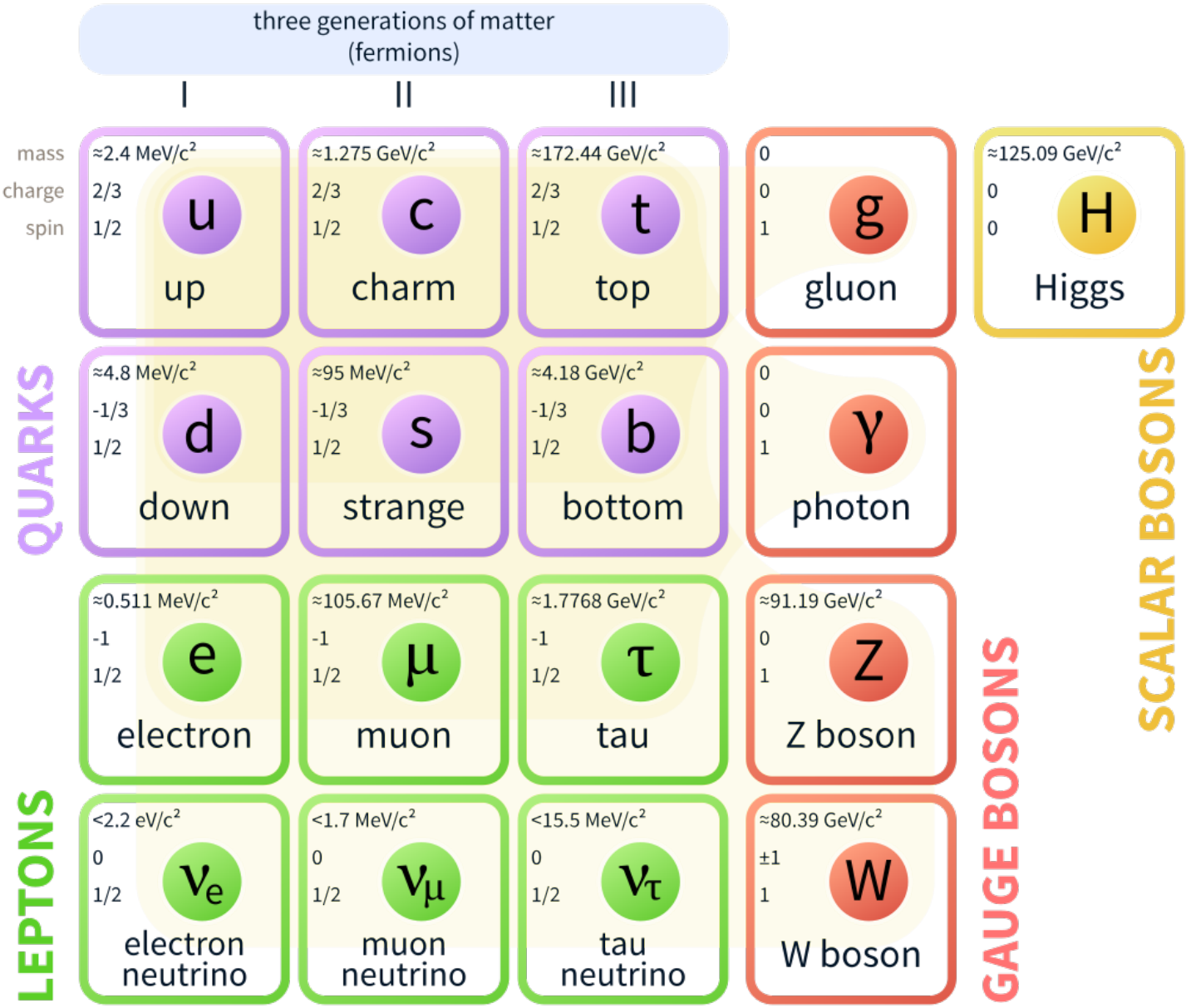
$$\equiv \cos\left(\frac{|\lambda(x)|}{2} \right) + i \hat{\lambda}(x) \cdot \vec{\tau} \sin\left(\frac{|\lambda(x)|}{2} \right) \begin{pmatrix} \psi_p(x) \\ \psi_n(x) \end{pmatrix}$$

- Theory of SU(N) groups enters Physics

$$U^* U = 1_{N \times N} ; \text{Special } det(U) = 1$$

- More generally, the theory of Lie (1900) groups; non-abelian gauge invariance

Standard Model of Elementary Particles



Courtesy Scholarpedia

The Standard Model

- All fermions chiral … left and right helicities have differing charges
- "Electroweak" theory spontaneously broken

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{EM}$$

- Quarks come in 3 "colour"s

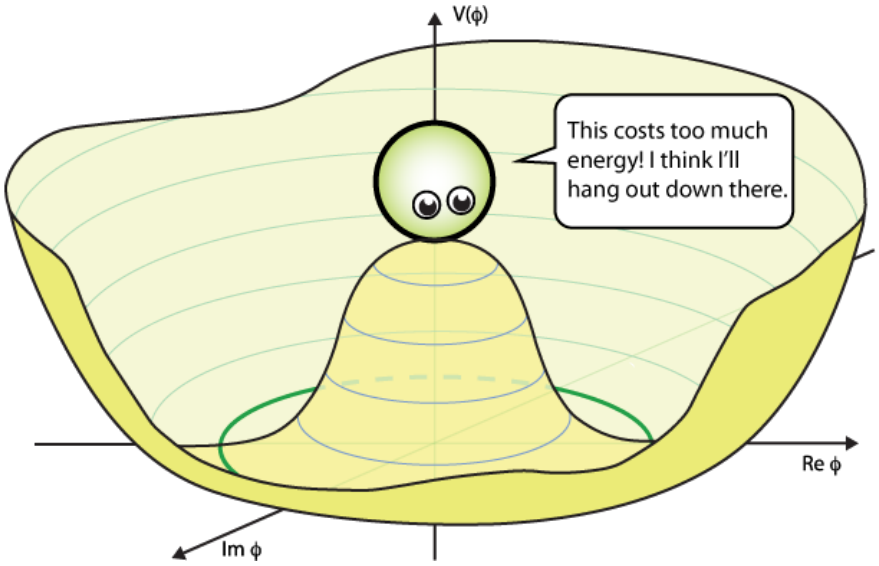
$$SU(3)_C$$

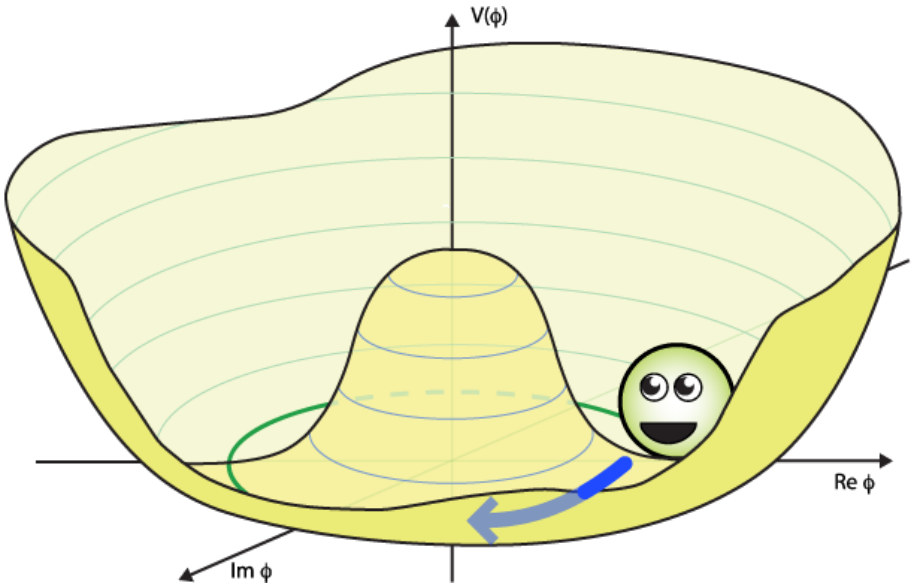
  – Quantum Chromodynamics; unbroken but "confined"

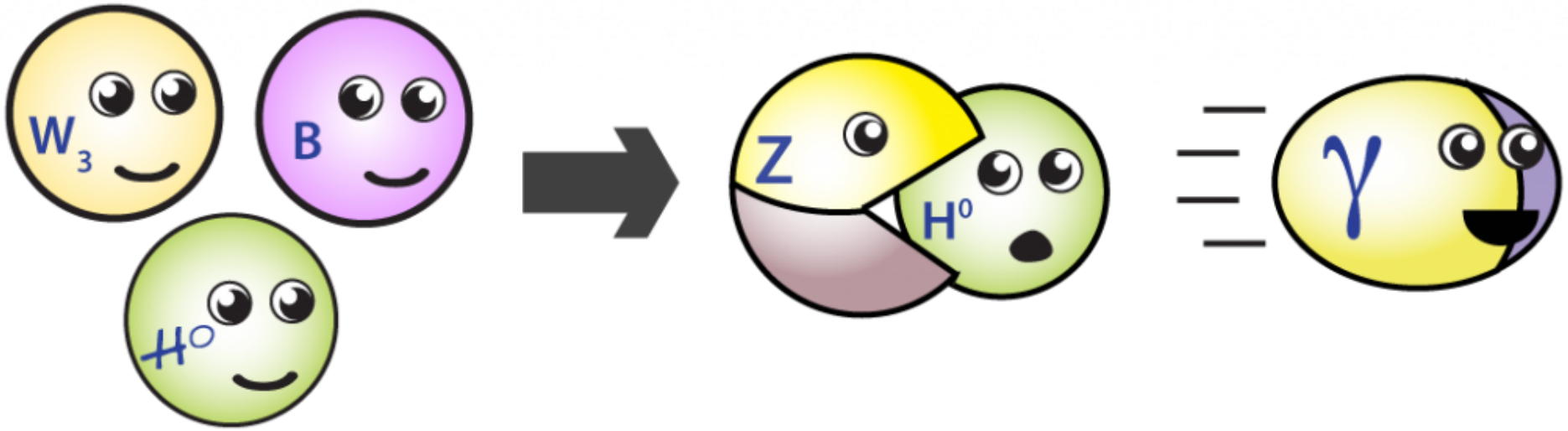
- All masses, fermions and gauge bosons, derived from "spontaneous symmetry breaking" with a massive scalar Higgs boson as the residual signature of this mechanism

The enigmatic Higgs boson

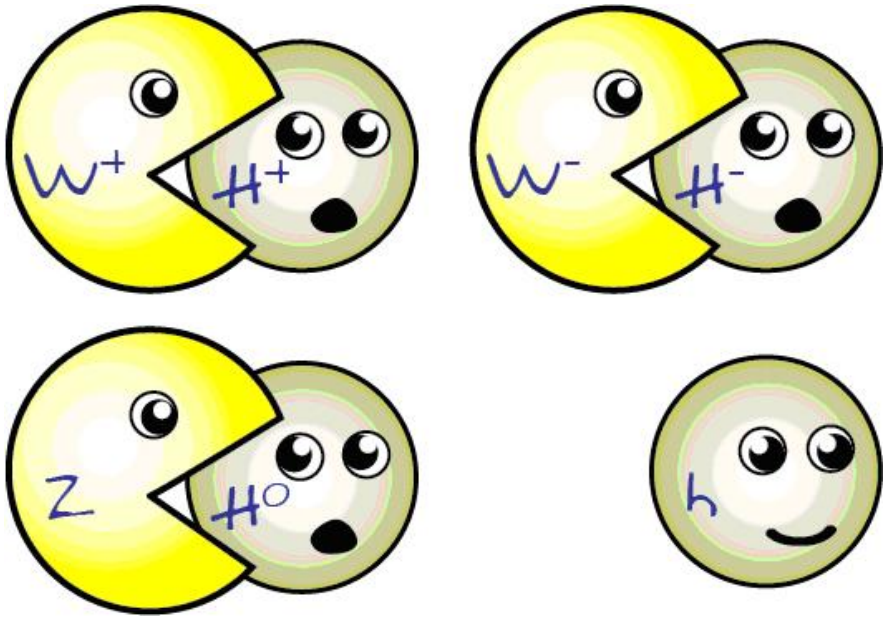
- If there is any higher mass / energy scale in the theory a scalar theory must necessarily obtain masses of that scale
- The Planck scale exists 10^{19} GeV
- Who ordered the 250 GeV scale?
  → How can it remain robust as stand alone?





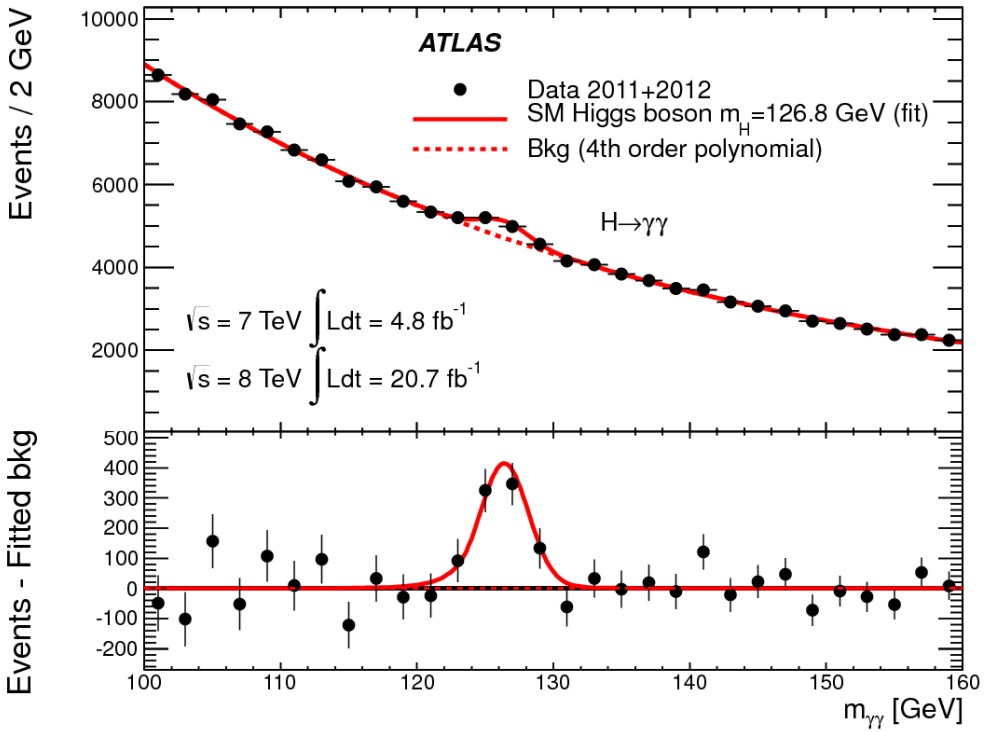


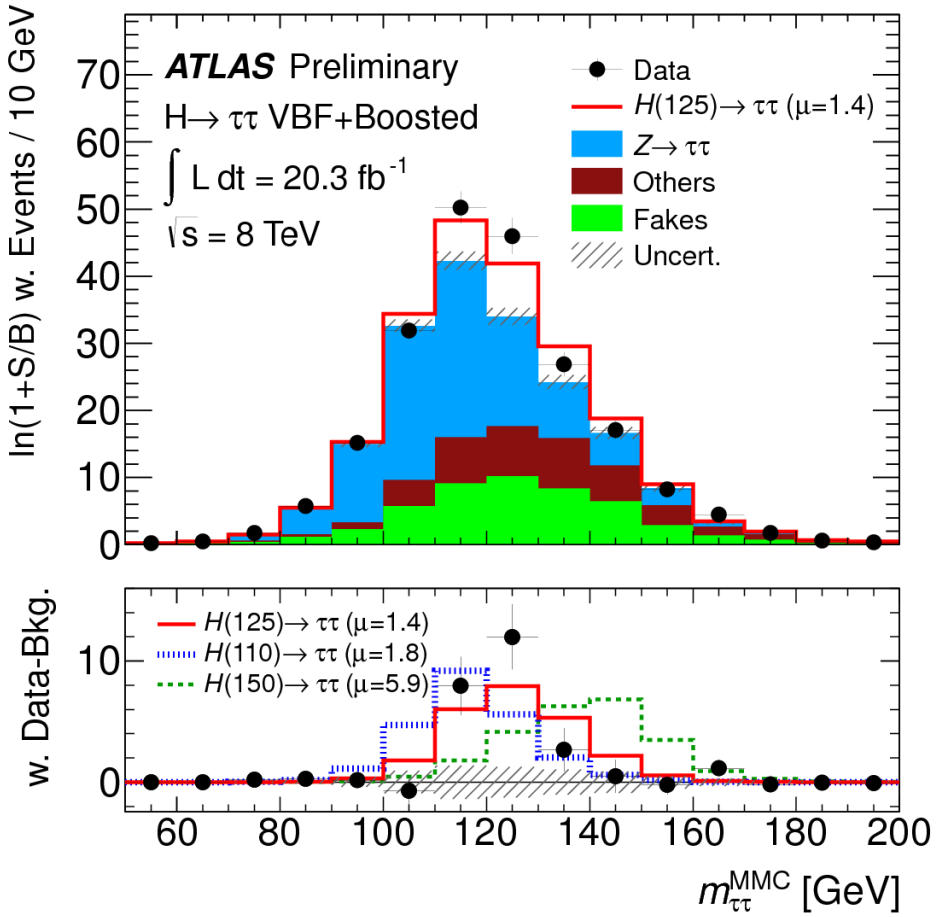
Higgs mechanism according to Pacman manual



Courtesy Quantum Diaries or Dennis Silverman

Higgs discovery



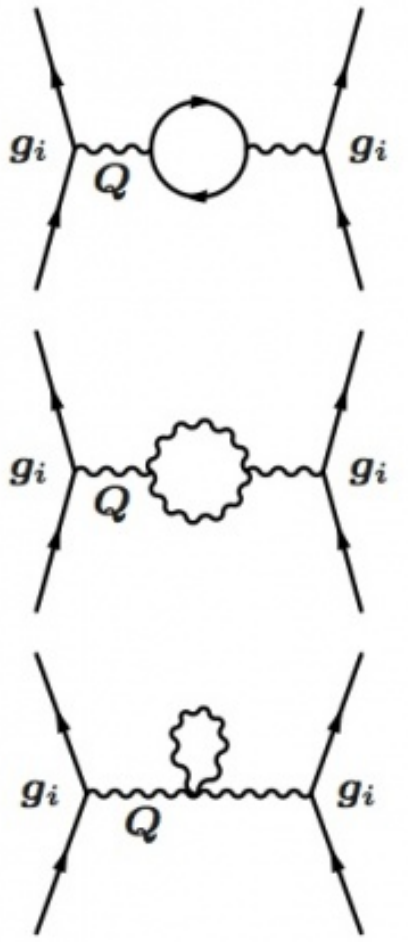


The “running” coupling constant

- Couplings are not constants
- The theory makes perturbative sense only if we agree to rescale the couplings with energy scale of the scattering experiment.

$$\frac{1}{\alpha_i(Q^2)} = \frac{1}{\alpha_i(M_Z^2)} - 4\pi b_i \ln \frac{Q^2}{M_Z^2}$$

i indexes the three couplings



Courtesy Scholarpedia

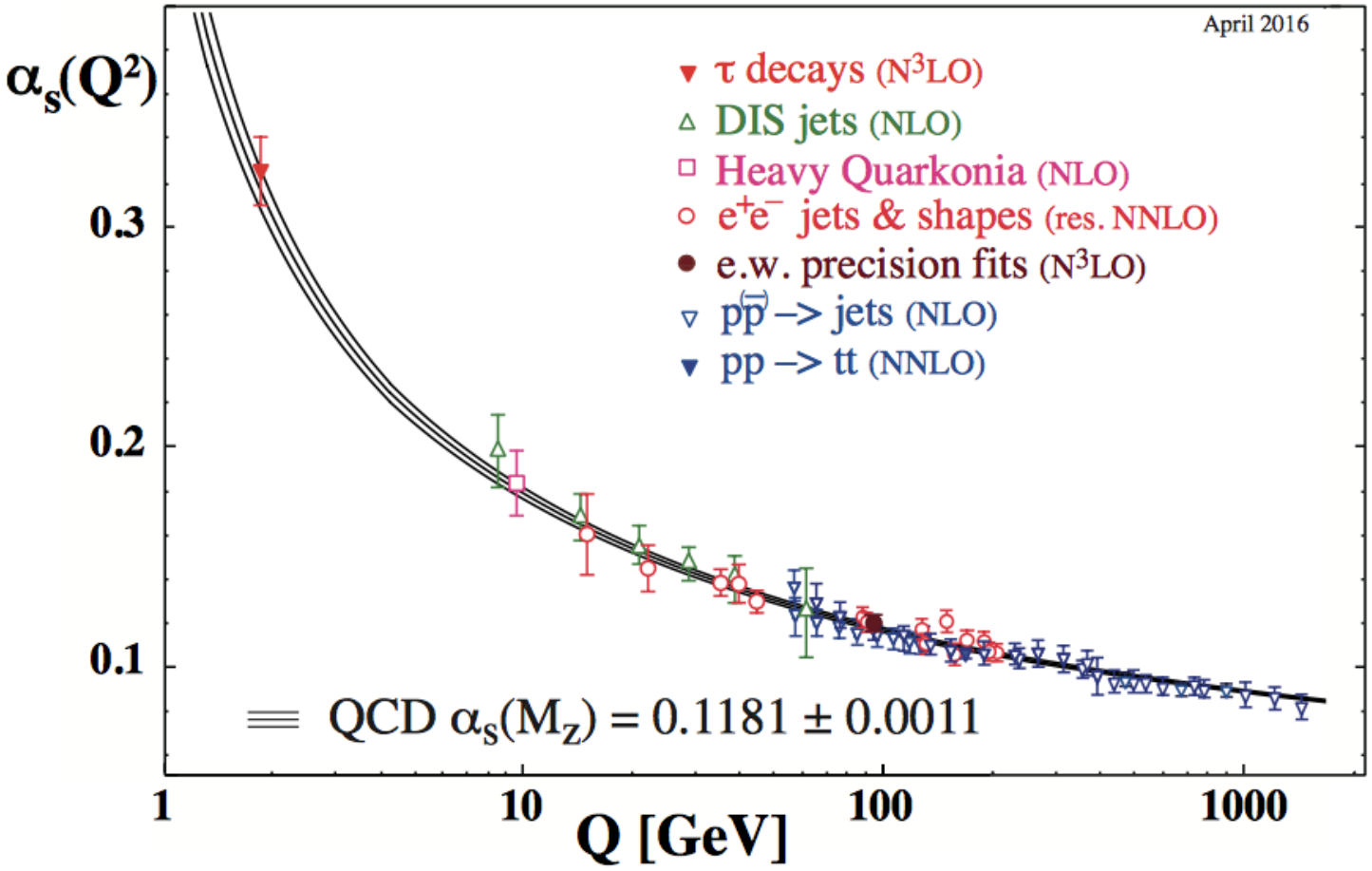
Asymptotic freedom

SU(3) of strong interactions -- weak at high scales

Strongly bound bound states at low energies

BE >> rest masses

Quarks and gluons can never be free



The galaxy of stalwarts

- Precursors : Fermi Wu Lee and Yang
- Yang and Mills
- Marshak and Sudarshan Feynman and Gell-Mann
- Schwinger
- Nambu Goldstone
- Gallery of Nobel winners …

  Glashow Salam Weinberg 1979 (SU(2)XU(1) model; 1962, 1967)

  Rubbia van der Meer 1984 (discovery of W's and Z 1983)

  't Hooft Veltman 1999 (for his 1971 – 72 papers)

  Gross Politzer Wilczek 2004 (for their 1973 papers)

  Englert Higgs 2013 (for their respective 1965 papers)

Glashow

Salam

Weinberg

Rubbia

van der Meer

‘t Hooft

Veltman

Gross

Politzer

Wikzec

Englert

Higgs

Some Quantum Field Theory theorems - 2

"… Theorems -1" transparency was the running coupling constant

- Chirality adds the elegance of generating mass spontaneously
- Anomalous violation of fermion currents

  Only for special choices of Fermion representations does the theory remain perturbatively meaningful
- Anomaly : a constraint on theories … including the number of dimensions in String Theory

Some Quantum Field Theory theorems - 3

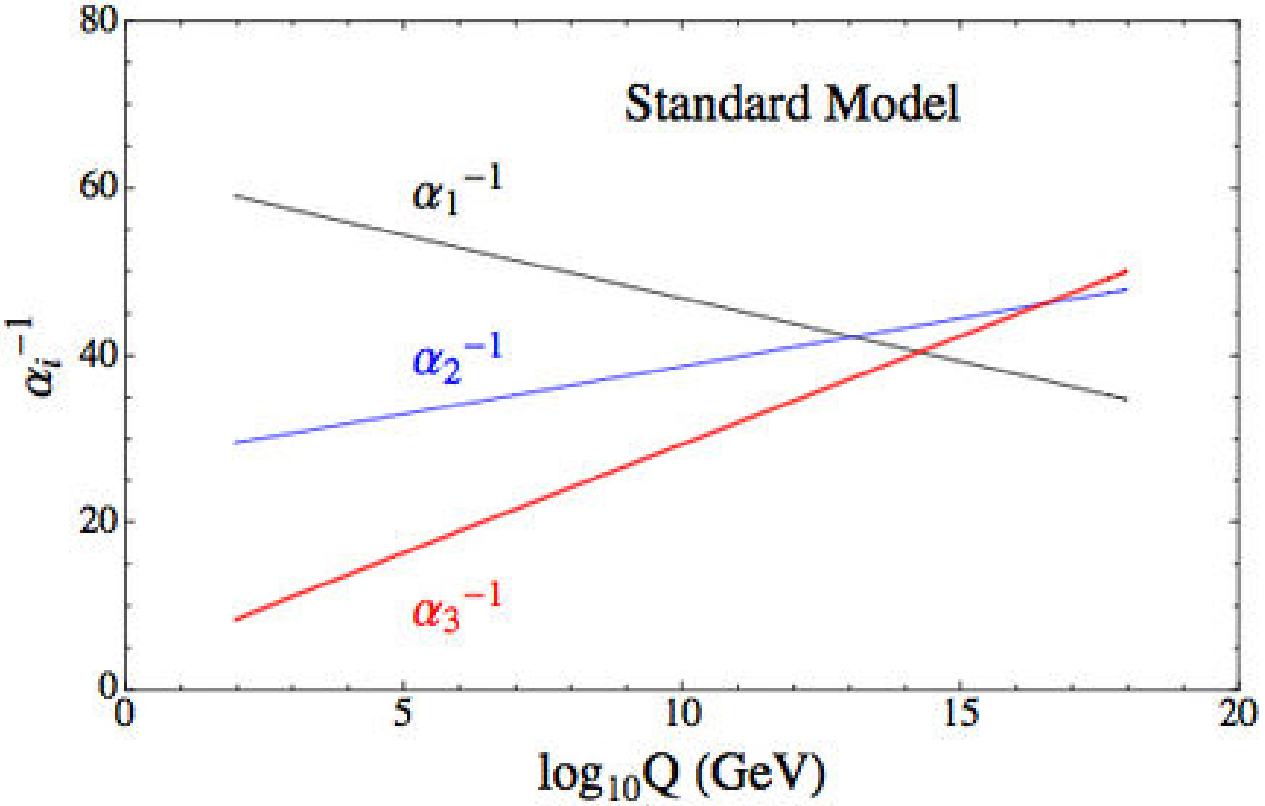
- Massless spin-1 particles have to couple to a conserved vector current (Electromagnetism)
- Massless spin-2 particles have to couple to a conserved rank-2 tensor

  → Energy-momentum tensor exists … Universality of energy makes this the unique rank 2 conserved tensor

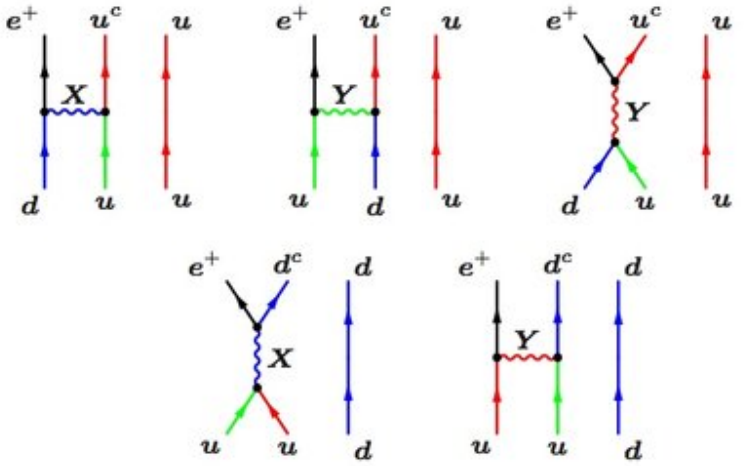
  → Uniqueness of Einstein Gravity
- Weinberg-Witten theorem :Strong constraint on obtaining Gravity as an effective theory from a “sensible” underlying theory.

Dreams of Grand Unification

Pati-Salam, Georgi Glashow; Georgi Quinn and Weinberg



← Missed opportunity for unification

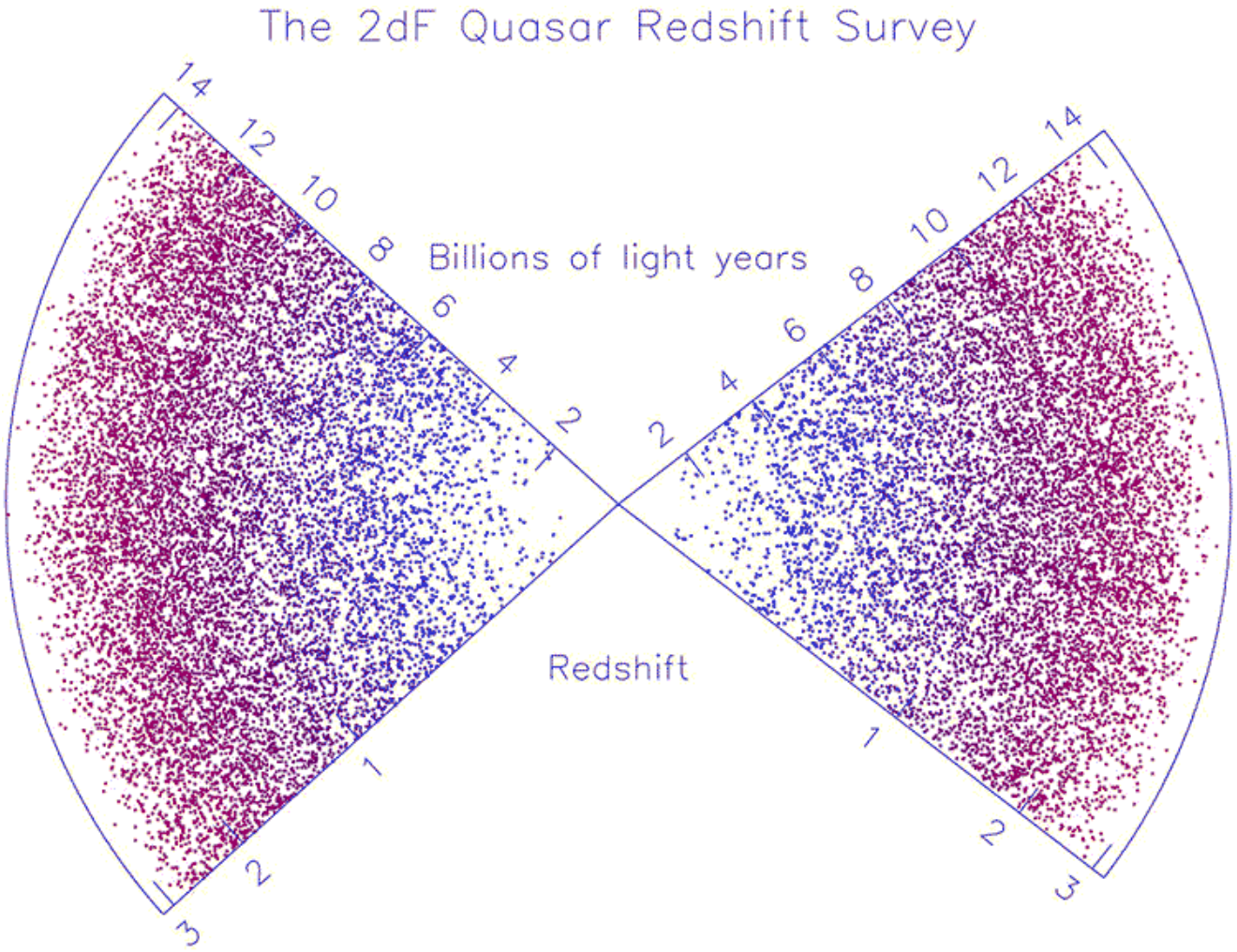


Potential signal as proton decay →

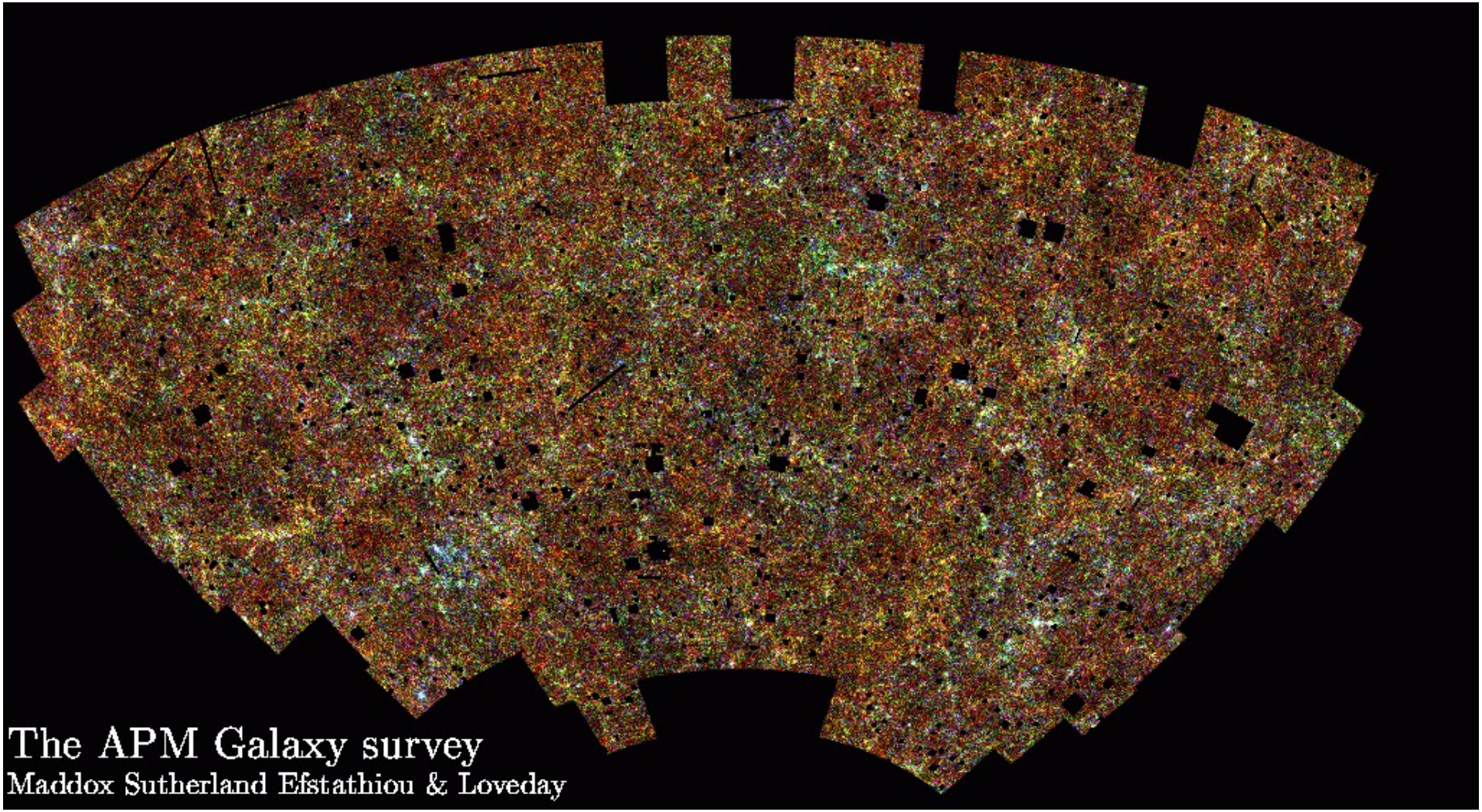
Part III

The incredible Universe

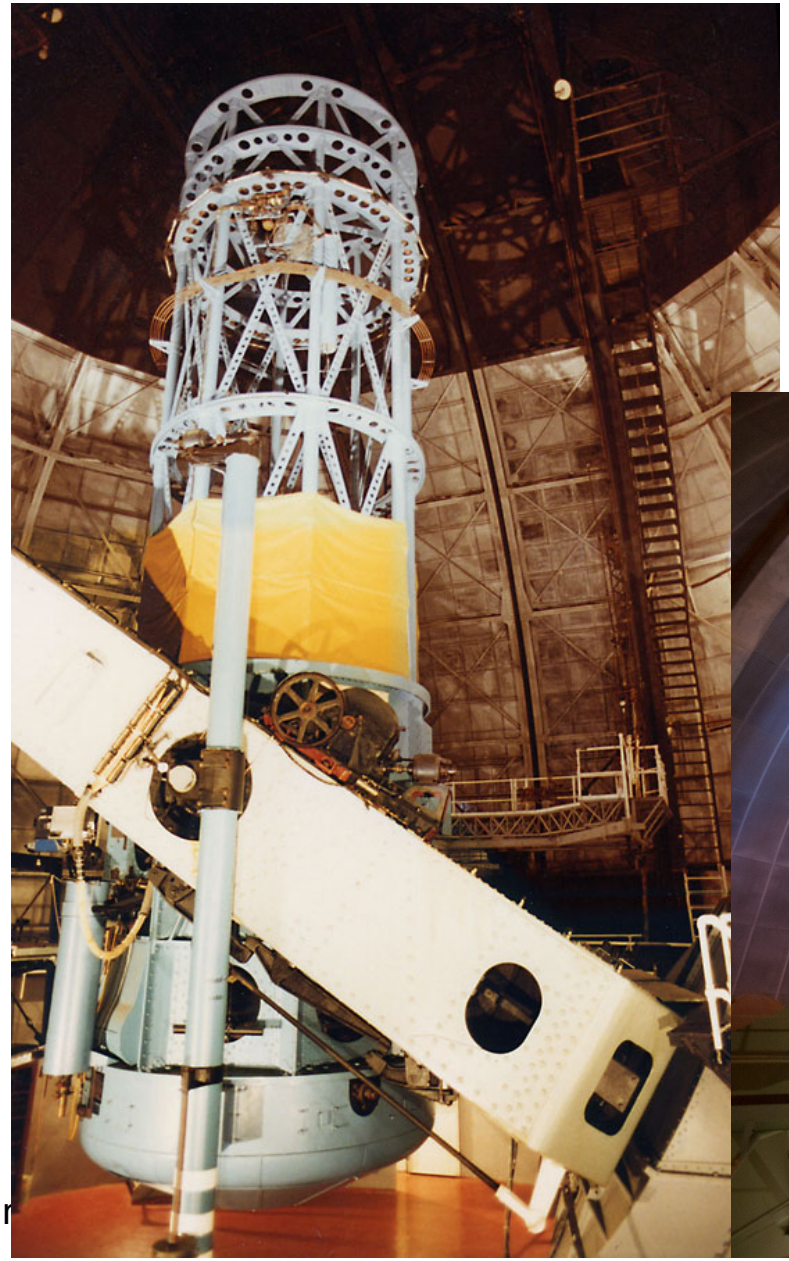
The Universe, observed



The Universe, observed



From metaphysics to Astrophysics



Construction of the large telescopes
Opened up the Universe to empirical scrutiny

Edwin Hubble draws the line (1929)

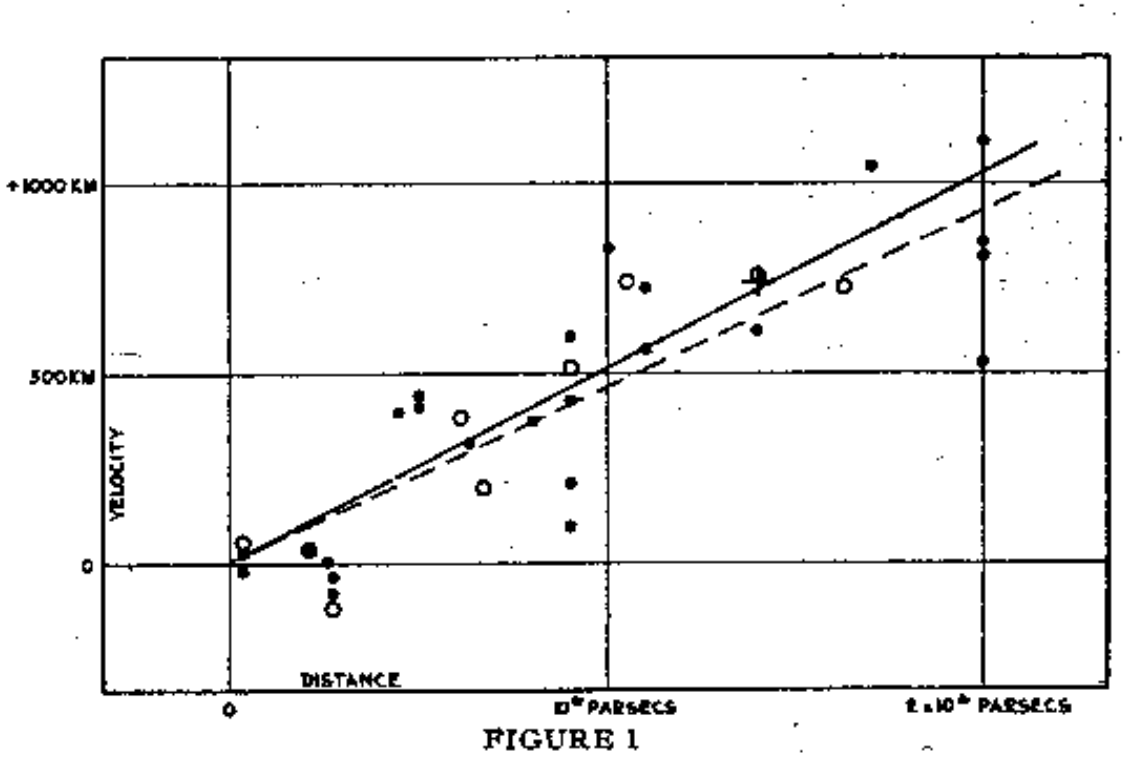


FIGURE 1

The three laws of cosmology

- Law 1: Assume an isotropic and homogeneous Universe

$$ds^2 = dt^2 - R(t)^2\{\frac{dr^2}{1+kr^2} + r^2 d\theta^2 + r^2 \sin^2 d\phi^2\}$$

k value -1, or 0 or +1 sets constant spatial curvature

- Law 2: Dynamics of R(t) determined by Einstein's equations

$$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 + \frac{k}{R^2} = \frac{8\pi}{3}G\rho$$

- Law 3: Provide equation of state

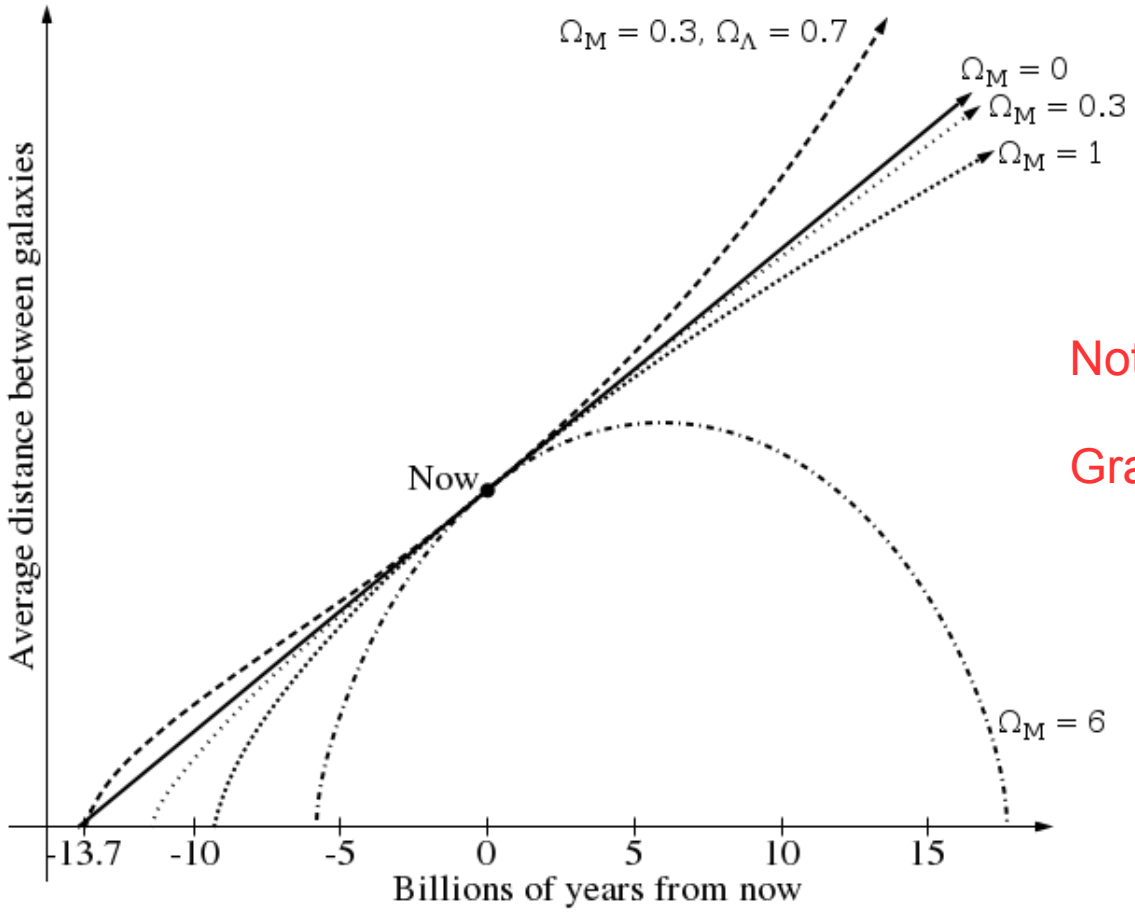
$$p = w\rho$$

Then energy conservation determines rho in terms of R.

Your Friedmann universe is ready to be solved for

Sample universes

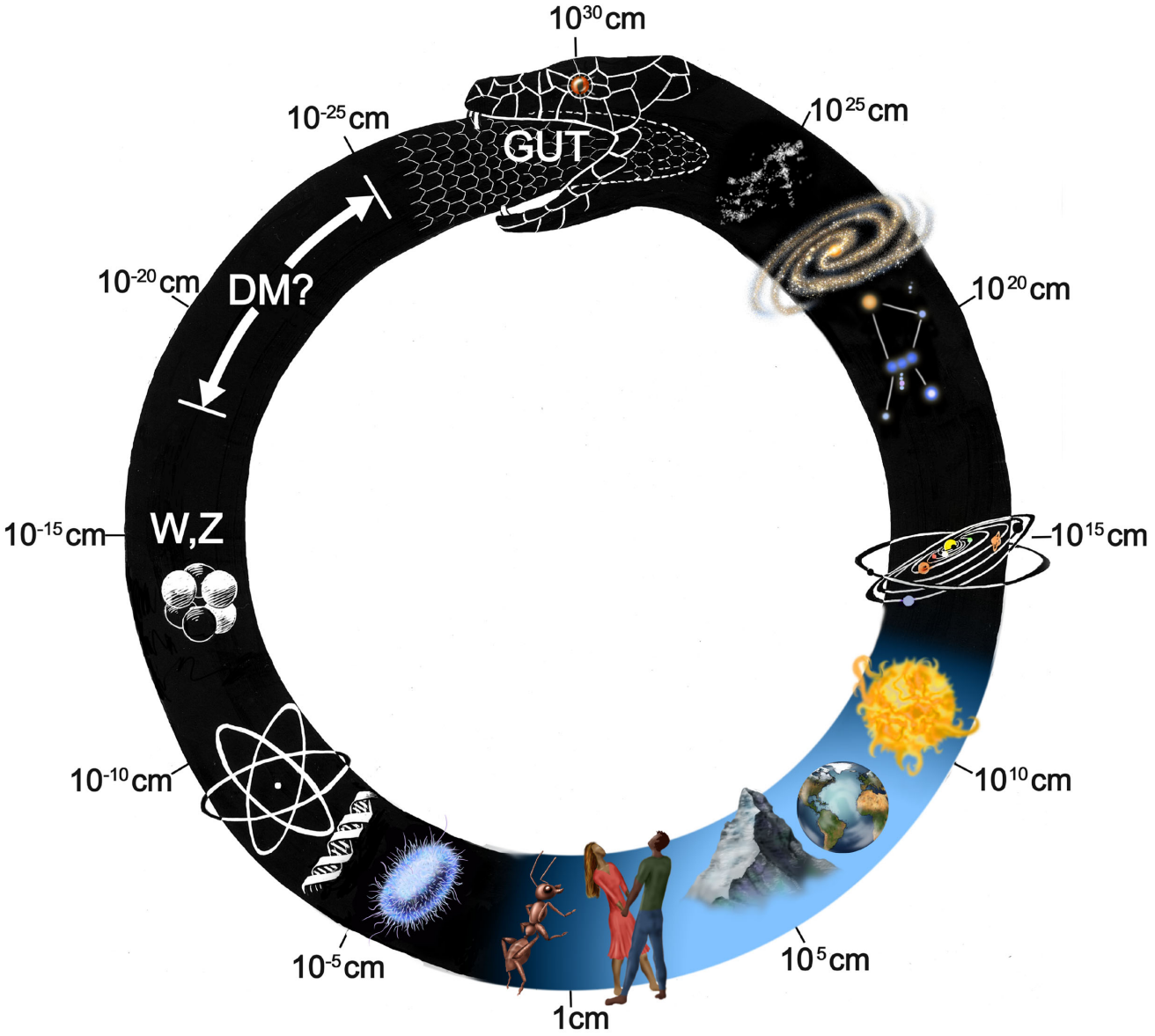
- A. A. Friedmann 1922-24



Note generic deceleration

Gravity only attracts

The cosmic Urobors – Glashow



Einstein is embarrassed by this cataclysmic inevitability

.... add a Λ (Einstein 1924) in the law for $R(t)$ to avoid expanding / contracting Universe.

$$H(t)^2 + \frac{k}{R(t)^2} - \Lambda = \frac{8\pi G}{3}\rho(t)$$

✔ This introduces a new fundamental constant of nature, of dimensions $[L^{-2}]$,the Cosmological Constant

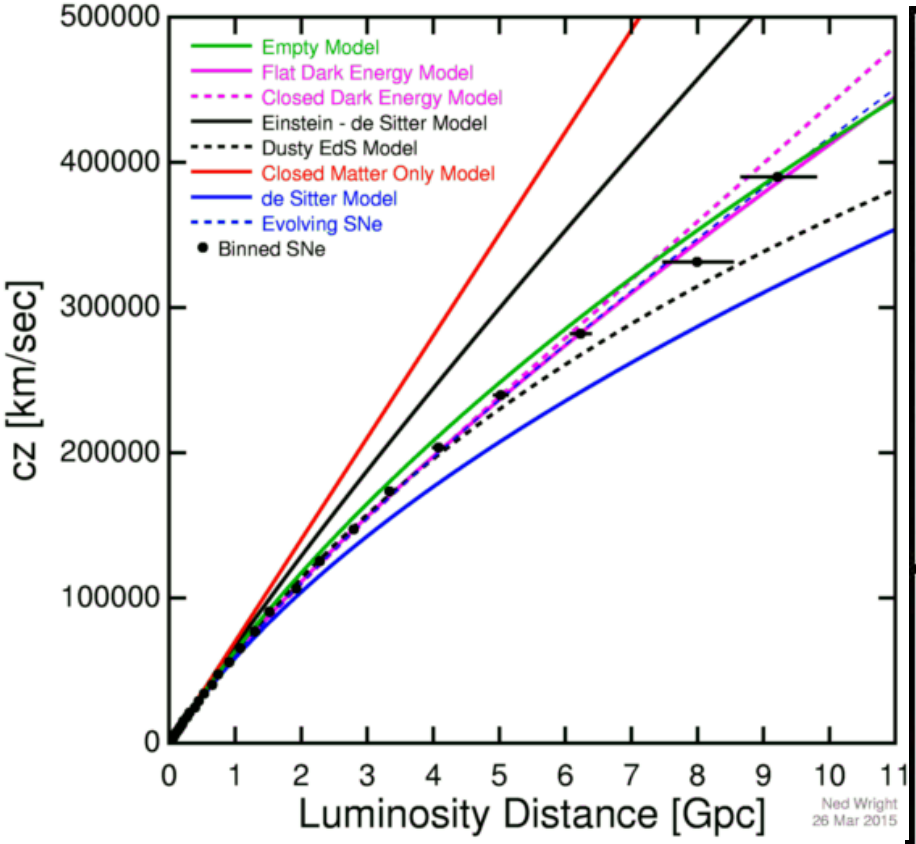
✔ If the $\Lambda$is transferred to the right hand side, it looks like a contribution to ρ, satisfying the unusual equation of state $p = -\rho$.

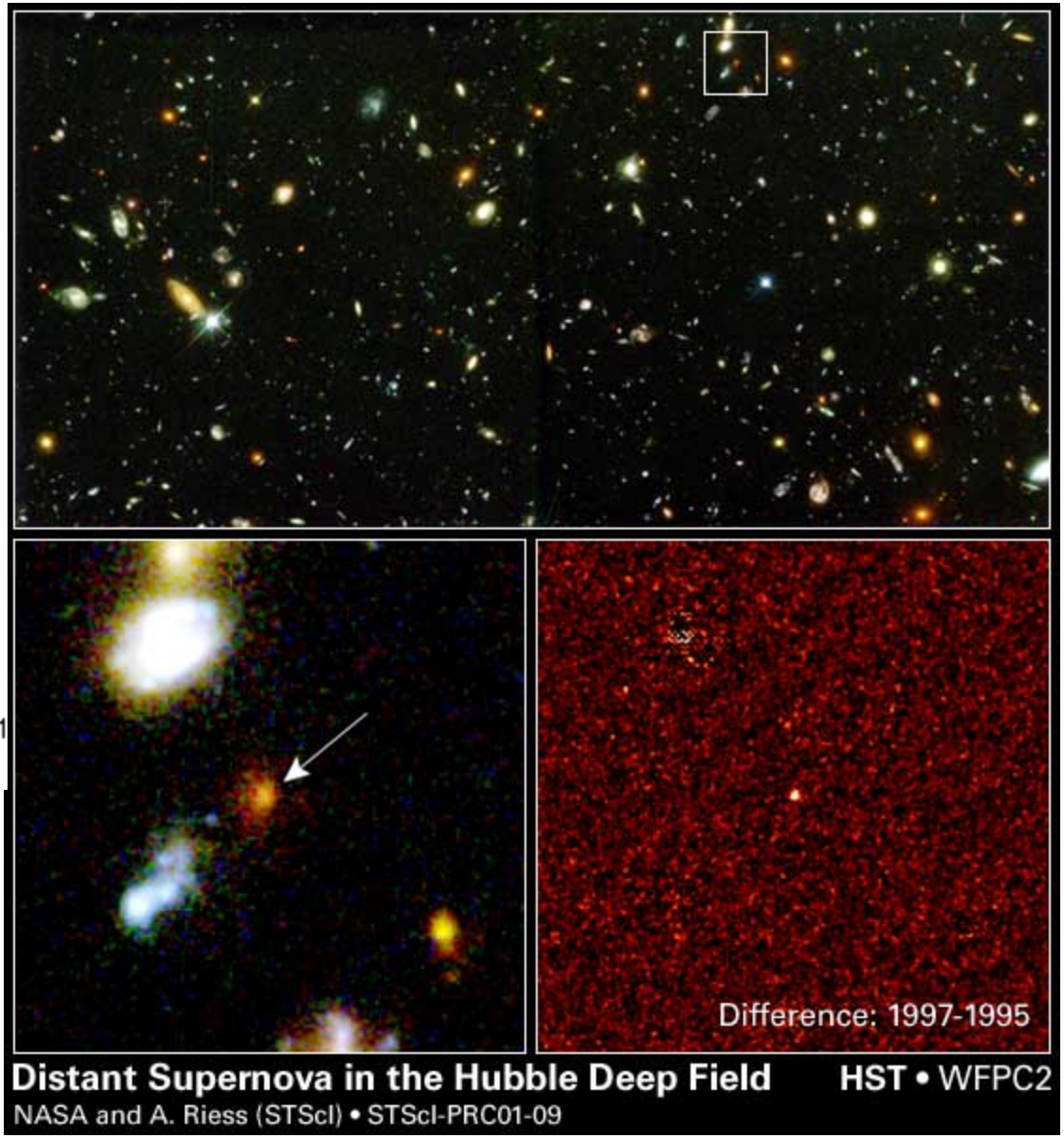
Until Hubble discovered the expansion. Then Einstein wrote to a colleague “away with it if it is not required”.

Vacuum energy and the Higgs mechanism

- The Higgs potential is a nontrivial polynomial
- We are assuming that its ground state energy is zero
- Gravity sees all forms of energy and redefinition of zero of energy not possible
- Current energy density of the Universe is $\rho_0 \sim 10^{-46} (GeV)^4$ (in natural units)
- So Higgs energy $V_0 \sim 10^{10} (GeV)^4$ has to be set to zero …. to one part in 10^{56}
- Theoretical solution → invoke Supersymmetry!!

A blast from the remote past





- Type Ia supernova as new standard candles
- Push back distance scales to 7 Bpc
- Imply accelerating Universe

The problem of vacuum energy

- Quantum Field Theory mechanism “generates” undetermined amount of vacuum energy
- Standard Model has energy density $V_0 \sim 10^{10} (GeV)^4$ to be adjusted
- If we pursue Grand Unification there would be corresponding Higgs mechanism with residual energy density $V_0^{GUT} \sim 10^{60} (GeV)^4$
- Suppose find an elegant mechanism to set these to zero
- Now we need to generate a residue of order $\rho_\Lambda \sim 10^{-46} (GeV)^4$ designated the Dark Energy density

As I was going up the stair,
I met a man who wasn't there,
He wasn't there again today,
I wish, I wish he'd stay away

Hugh Mearns quoted in Weinberg RMP 1989

A menagerie of theories

- Kaluza – Klein … embed Yang-Mills in higher dimensional General Relativity 1925; 1936
- Quantum Field Theory of Gravity
- Supergravity based on the Lie group $E_8 \otimes E_8$
- Superstring Theory Subsumes the above
  – Needs 10 space-time dimensions for quantum consistency
- Extra dimensions with branes at fixed points
- Conformal invariance → all masses and scales emergent

In Conclusion

- A synthesis or Electromagnetism and Weak nuclear force, into Electroweak Theory has been achieved in the gauge group $SU(2)_L \otimes U(1)_Y$
- The Strong force follows exactly the same pattern but with the group $SU(3)_C$
- Gauge principle has uncanny analogies to General Covariance obeyed by Gravity
- The need for spontaneous symmetry breaking results in vacuum energy problem
- We have difficulty understanding the minuscule vacuum energy driving today's Universe into accelerated expansion.

Thank You !!