Einstein Lecture Series

Nehru Planeterium

Creation of the General Theory of Relativity

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Creation . . . ?

Invention vs. Discovery

Facts exist, laws relate them

. . . an elegant formulation is truely

¡¡ an invention !!

Achievements of the General Theory of Relativity

- Relativistically correct theory of Gravitation !
- Program for creating "whole theories" !!

Local Symmetry

Dynamical Symmetry

Gauge Symmetry — Yang & Mills

$\Rightarrow$ Theory of Elementary Particles

Glashow, Weinberg, Salam

Special Theory of Relativity

Lorentz contraction, time dialation

$$c\Delta t' = \gamma(c\Delta t - \beta\Delta x)$$
$$\Delta x' = \gamma(\Delta x - \beta c\Delta t)$$

$$\beta \equiv v/c \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Invariance (covariance)
of Electromag. laws under change of inertial frames

Peculiar stretching and straining among atoms??

... Universals laws of kinematic transformations ...

Pythagoras

$$|\Delta\vec{x}|^2 = (\Delta x^1)^2 + (\Delta x^2)^2 + (\Delta x^3)^2$$

invariant under rotations

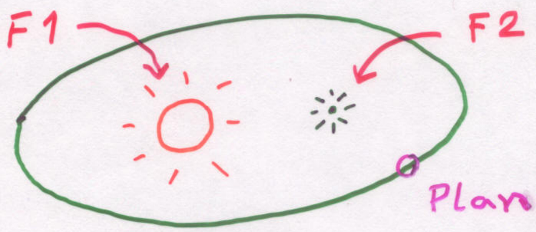
Einstein & Minkowski

$$\Delta s^2 = (c\Delta t)^2 - (\Delta x^1)^2 - (\Delta x^2)^2 - (\Delta x^3)^2$$

invariant under rot. + Lorentz "boosts"

Fascinating Gravity

The Heavens in synchronised swimming !!



Kepler's laws

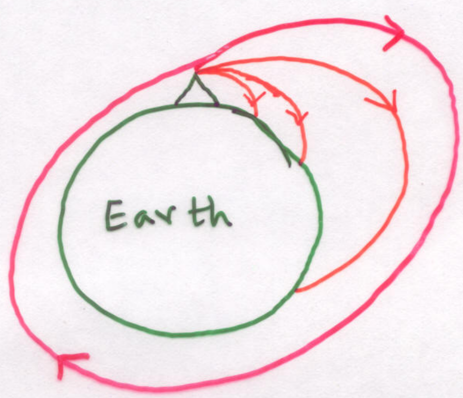
- Orbits ellipses, Sun at one Focus
- Equal areas in equal times
- $GM \propto \omega^2 a^3$ (1-2-3 law)

Newton : ($1/r^2$ law)

"Moon falls 0.0045 ft/sec towards the earth and is ~60 earth radii away"

"And an apple at 1 earth radius falls 16 ft in a second" !!!

Unification of the Heavenly with the Terrestrial Gravity



The Principle of Equivalence

... of inertial and gravitational masses

$$m\,a = F = \frac{q_1 q_2}{[4\pi\epsilon_0]}\,\frac{1}{r^2}$$

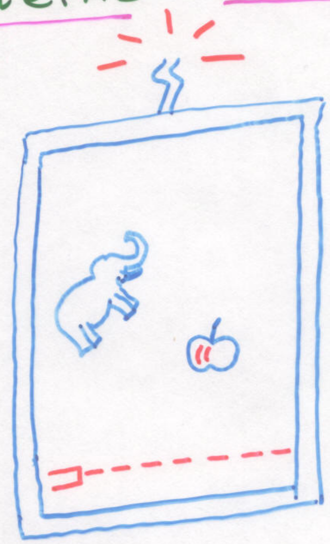
But

$$m\,a = F = G\,m\,m_2 \times \frac{1}{r^2}$$

inertial mass

Gravitational "charge"

Einstein's Elevator



In sufficiently small regions of spacetime, frames of reference exist in which effects of Gravity disappear.

Bending of light

The Geometry connection:

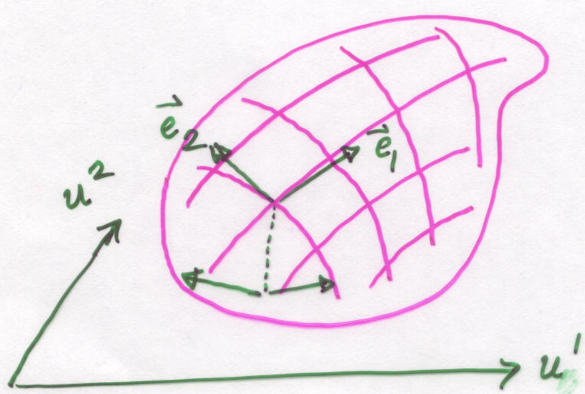
Gravitational trajectories are a property of the space-time

Spacetime is itself curved

Equivalence Principle $\Longrightarrow$ Riemannian Geometry

Differtial Geometry: [Gauss, Riemann, Levi-Civita]

Study of "curved" spaces using methods of differtial calculus



$\vec{e}_1 \cdot \vec{e}_2$ (Pythag.)

$$= e_1^x e_2^x + e_1^y e_2^y + e_1^z e_2^z$$

$\vec{e}_1 \cdot \vec{e}_2$ (Gauss)

$$= g_{xx} e_1^x e_2^x + g_{yy} e_1^y e_2^y + g_{zz} e_1^z e_2^z + 2 g_{xy} e_1^x e_2^y + 2 g_{yz} e_1^y e_2^z + 2 g_{zx} e_1^z e_2^x$$

Tensors:

$\{\text{scalar}\}$, $\{\text{vector}\}$, $\left\{\begin{matrix}\text{tensor}\\ \text{rank } 2\end{matrix}\right\}$, $\left\{\begin{matrix}\text{tensor}\\ \text{rank } 3\end{matrix}\right\}$, ...

tensor rank 0 — tensor rank-1 — $T^{(2)}$ — $T^{(3)}$

$s \cdot v = v$ $\quad s \cdot s = s$ $\quad (v \cdot v) = s$

Let $v \otimes v$ be such that

$$(v \otimes v \cdot v) = v \otimes (v \cdot v) = v \otimes s = v$$

So $(v \otimes v \cdot v) = v$ $\quad$ Remember $\left(A \right)\left(x \right) = \left(y \right)$

And so on $v \otimes v \otimes v$

Important property : linearity

$$\left(v \otimes v \cdot (a v_1 + b v_2)\right) = a\left(v \otimes v \cdot v_1\right) + b\left(v \otimes v \cdot v_2\right)$$

Examples:

1. Matrix

$$\left(A \right)\left(x \right) = \left(y \right)$$

$$T^{(2)} \cdot v = v$$

$$\left(w \right)\left(A \right)\left(x \right) = s$$

Note: Dimension of v being n, dim of $T^{(2)}$ will be $n \times n$, $T^{(3)}$ $n \times n \times n$ etc

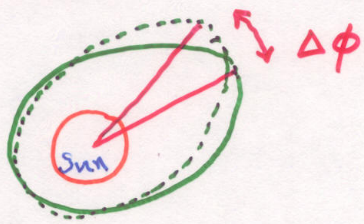
2 Metric tensor

$$\Delta s^2 = \sum_{\mu,\nu=0}^{4} g_{\mu\nu} \Delta x^\mu \Delta x^\nu$$

$$(T^{(2)} \cdot v) \cdot v = s$$

Predictions (1911 - 1913)

1. Precession of perihelion of Mercury

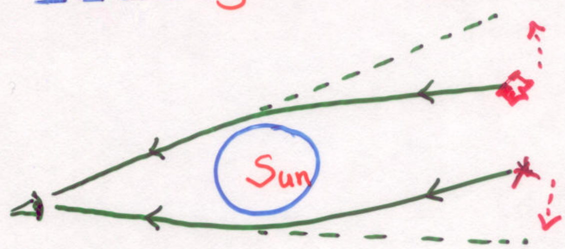


43'' of arc/century

Exactly predicted

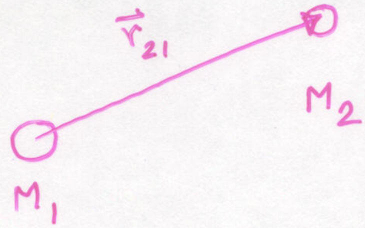
unsolved problem of 50 yr.s

2. Bending of light



3. Redshift of light

Equations for the Gravitational field?



$$F \propto \frac{1}{|\vec{r}_{21}|^2}$$

law is instantaneous

Generalise Newton's Field equation?

$1/r^2$ is a solution of the equation

$$\nabla^2\phi = 4\pi G\rho$$

mass (inertial) density

Grav. F. $= -\vec{\nabla}\phi$

[Grav. Force = $m_{test} \times$ G.F.]

Try Relativistic:

$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi \overset{??}{=} 4\pi G\rho$$

Scalar under Lorentz transf.?

Lorent tranf. involves momentum flux

Nordstrom's vector theory

Einsteinian Dilemma

"The sought after generalisation will surely be of the form

$$\Gamma_{\mu\nu} = \kappa T_{\mu\nu},$$

where κ is a constant and $\Gamma_{\mu\nu}$ is a contravariant tensor of second rank that arises out of the fundamental tensor $g_{\mu\nu}$ through differential operations ... ...it proved impossible to find a differential expression for $\Gamma_{\mu\nu}$ that is a generalisation of [Poisson's] $\nabla^2\phi$, and that is a tensor with respect to arbitrary transformations ... ... It seems most natural to demand that the system be covariant against arbitrary transformations. That stands in conflict with the result that the gravitational field does not possess this property."

[A. Einstein and M. Grossmann, 1913]

Source of the dilemma:

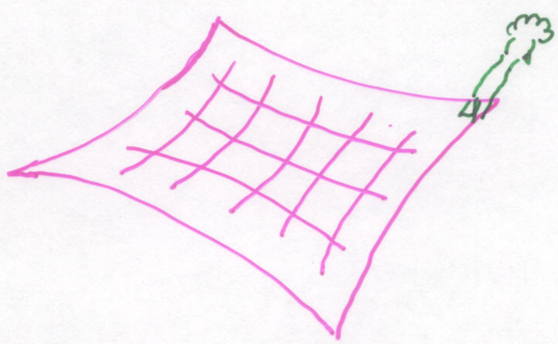
- Gravity decides space-time properties
- Gravity carries energy

Can "space-time" have "energy"?

If energy goes into $T_{\mu\nu}$, L.H.S. is not a tensor ...

Strong Principle of Equiv, OR

$\boxed{\text{No prior geometry}}$



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

self energy of Gravity on L.H.S.!

No local density of Grav. field energy!!

Source-flux Theorems & conservation

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}\cdot\vec{j} = 0$$

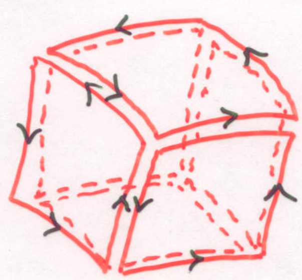
$$\Rightarrow \quad \frac{d}{dt}\int_V \rho = -\oint_{\substack{\text{Boundary}\\ \partial V}} \vec{j}\cdot d\vec{A}$$

$$\frac{\partial}{\partial x^0}T^{0\nu} + \sum \frac{\partial}{\partial x^i}T^{i\nu} = 0 \qquad \nu = 0,1,2,3$$

is of this form

... modifies to include grav. energy of matter particles

What about $\text{"}\frac{\partial}{\partial x^0}\text{"}G^{0\nu} + \sum \text{"}\frac{\partial}{\partial x^i}\text{"}G^{i\nu} = 0$??



Contrib. of every face counter-clockwise looking from outside

"Boundary of a boundary is Zero"

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